

Predicting the Risk Level of Guerrilla Heavy Rainfall by Using the Quantitative Risk Prediction Method with Multiple Doppler Radar Analysis

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Synopsis

Japan has suffered from devastating flood disasters caused by localized heavy rainfall known as Guerrilla heavy rainfall recently. For reducing the damage, it is necessary to predict the risk of GHR precisely. So, we aim to propose an accurate quantitative risk prediction method. One of the importance was that the relationship between the predicted risk level and the variables was considered depending on each rain stage because the variables showed different characteristics according to the development of the convective cloud. The other one was that the variables were estimated with real wind field data by multiple Doppler radar analysis. Then, the multilinear regression was used for finding the correlation between the predicted risk level and the variables with accuracy. The accuracy of multilinear regression was estimated by a Receiver Operating Characteristic analysis. As the result, the most appropriate regression among the relevant variables was composed of reflectivity, vorticity, divergence, and updraft by multiple Doppler radar analysis. It is possible to predict the risk quantitatively with high accuracy of 90% at the early rain stage.

Keywords: Guerrilla heavy rainfall, Quantitative Risk Prediction, Multiple Doppler Radar Analysis

1. Introduction

Recently, localized severe heavy rainfalls, which have not been experienced in the past, have frequently occurred in Japan due to the effects of climate change (Miyasaka et al., 2020). In particular, the Guerrilla heavy rainfall (abbreviated as GHR), which is a rapidly growing isolated single cumulonimbus when water vapor rises to the upper atmosphere with the ascending air current, could trigger flash floods in a small river basin and caused extensive economic damages and casualties. In 2008, more than fifty people have washed away and five people were killed by a tragic flash flood caused by GHR in Toga River, Kobe city. It is quite difficult to

observe and predict the rapidly developed rainfalls. If the risk of heavy rainfall can be identified and alerted in advance, it would be possible to safely evacuate the danger. Nakakita et al. (2014) developed qualitative risk prediction methods based on the early detection of the rain cells aloft. These methods can predict the risk of the localized heavy rainfall qualitatively and track the development of convective cells by considering all rain stages. Following this previous research, we use the risk of terminology to represent the severity of the localized heavy rainfall intensity.

To discriminate the risk quantitatively, we aim to develop the early detection and quantitative risk prediction method by considering the performance of variables that have different characteristics

depending on each rain stage. Also, we deal with the vertical vorticity, reflectivity, divergence, and updraft to find the correlation with the predicted risk level. Especially, the vorticity, divergence, and updraft are important variables at the beginning of an isolated single cumulonimbus cloud (Ulanski and Garstang, 1978). The vorticity and divergence can be estimated by using Pseudo radar analysis and multiple Doppler radar analysis. On the other hand, the updraft can be calculated by multiple Doppler radar analysis only because the analysis can retrieve a three-dimensional wind field. Pseudo radar analysis uses only a single Doppler radar observation which is a radial component of wind field that flows toward or away from the radar. To predict the risk level more accurately, it is necessary to estimate the three-dimensional wind field by using multiple Doppler radars. Therefore, in this study, we aim to propose an accurate quantitative risk prediction method. The objectives of this study are to (1) compare the total stage regression at all rain stages with the stage dependent regressions at each rain stage; (2) determine the multilinear regression with the highest accuracy by finding the best combinations of the variables (i.e. the reflectivity, vorticity, divergence, and updraft) with Pseudo and multiple Doppler radar analysis.

2. Methodology

2.1 The process of Guerrilla heavy rainfall occurrence

In Japanese media, the localized heavy rainfalls are denoted as “Guerrilla heavy rainfalls”. Fig. 1 illustrates the process from rain-cell into heavy rainfall, which is classified into the development, mature, and dissipation stages. In the early development stage, water vapor in the lower atmospheric layers rises and condenses to generate a convective cloud in an atmospheric instability condition. Cloud particles, however, are too small to be detected by weather radars before precipitation particles are formed. The detected particles can be defined as the first echo or a baby rain-cell. When the baby rain-cell is detected, a first rain stage is assigned. The rain stage is the process of convective cloud development. Then,

the baby rain-cell develops gradually, and the height of the cloud increases. However, any raindrop is not observed on the ground at this stage. In the mature stage, the volume of clouds increases in an ascending air current and finally falls with the descending air current. In the dissipation stage, as the ascending and descending air currents are weakened, the rainfall intensity decreases.

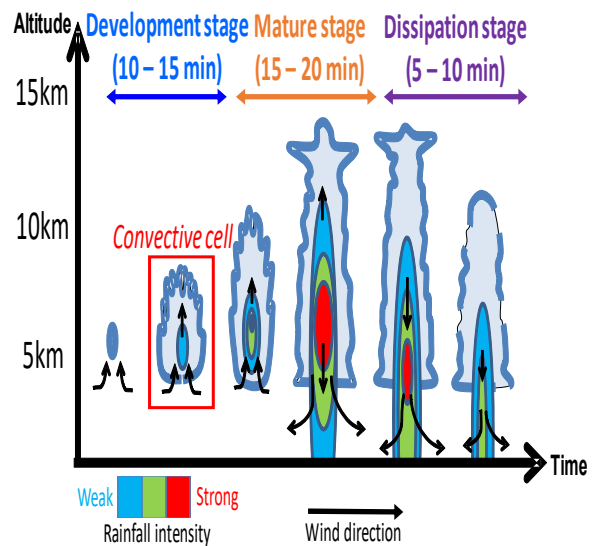


Fig. 1 The process of Guerrilla heavy rainfall occurrence.

2.2 The early detection of convective cells

Early detection means to find out the existence of the first echo in a convective cell on the upper atmospheric layer. At the first rain stage, a volume threshold, ranging from 0.125 km³ to 62.5 km³, is utilized to search convective cells. The initial convective cells can be identified by thresholds of 20 dBZ of radar reflectivity and vertical component of vorticity. The vertical vorticity is a measure of the local rotation of the flow. There are two ways to calculate vertical vorticity. One is Pseudo radar analysis; the other is multiple Doppler radar analysis. The Pseudo vertical vorticity is roughly estimated by applying the method proposed in Nakakita et al. (2017) with the radial velocity. From the radial velocity, the vorticity can be calculated by the following formula.

$$\zeta = (V_2 - V_1)/\Delta X, \quad (1)$$

where V_1 and V_2 (see definition in Fig. 2) are moving toward and away from the radar in the Cartesian coordinates and ΔX is the distance between the center of the mesh. Because the volume scanning observation using single Doppler radar could not measure horizontal wind velocities but the radial velocity. If we estimated the horizontal wind velocities by multiple Doppler radar analysis, the vertical component of vorticity can be calculated by the following formula.

$$\zeta = \partial v / \partial x - \partial u / \partial y, \quad (2)$$

where (u, v) are the horizontal wind velocities in the Cartesian coordinates. Also, Nakakita et al. (2017) proposed that the behavior of the vorticity is related to the growth of rain-cell. However, not every baby rain-cell aloft develops into severe heavy rainfall. Therefore, it is important to track the detected baby rain-cell prone to generate dangerous heavy rainfall. By using the early detection method, Nakakita et al. (2013) have clarified that the first echo was detected about 25 minutes on average before the maximum rainfall near the surface occurred, and Katayama et al. (2015) developed the qualitative risk prediction system which is operationally used in the Kinki region in Japan.

2.3 The multiple Doppler radar analysis

The multiple Doppler radar analysis could retrieve the three-dimensional wind field. Shimizu (2012) emphasized that this analysis can improve the understanding of the physical mechanisms behind heavy rainfall and forecasting hazardous weather. A variational method for the retrieval of the three-dimensional wind field was developed. To allow the measurement error, the Doppler velocities as weak constraints and the continuity formula as strong constraints are used in the cost function. The weak constrain appropriately accepts the observation errors from the radar data or assumptions in formulas. On the contrary, the strong constraint demands all of the analyzed and retrieved variables satisfy the formula exactly. Since the collected radar observation data could be affected by noise, in the Cartesian coordinates the

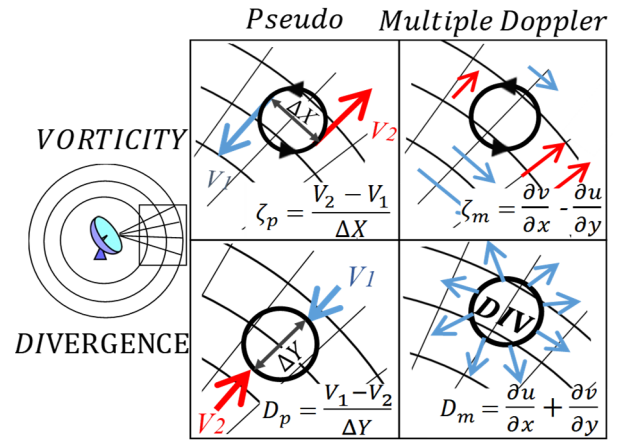


Fig. 2 Conceptual diagram of calculating vorticity and divergence by Pseudo and multiple Doppler radar analysis.

horizontal wind components (u, v) are used as weak constraints. By using the continuity formula, the vertical wind component (w) in multiple Doppler analysis is estimated. The continuity formula is expressed as follows:

$$\partial u / \partial x + \partial v / \partial y + \partial w / \partial z = kw, \quad (3)$$

where (u, v, w) are the three wind components in Cartesian coordinates, $k = -\partial(\ln \rho) / \partial z$, and ρ is the air density. With the continuity formula, the vertical wind component (w) is calculated by a weighted average of upward and downward integrations. The variational method minimizes the cost function, which is the sum of discrepancies between observations and analyses with weighting matrices that depend on the strength of the constraint (Protat and Zawadzki, 1999). In summary, the procedures are to 1) set initial control variables (u, v) ; 2) estimate the w in formula (3); 3) calculate the gradient of the cost function concerning control variables (Powell, 1977); 4) finish if the predefined condition of cost function was satisfied; otherwise, set new control variables and return to the step 2. Additionally, Ulanski and Garstang (1978) have proved that the updraft, downdraft, an upward flux of water vapor, maximum divergence, vorticity, and rainfall are presented in the convective cloud. So, the variables are collected to find the relevance with the risk level of localized heavy rainfall including GHR.

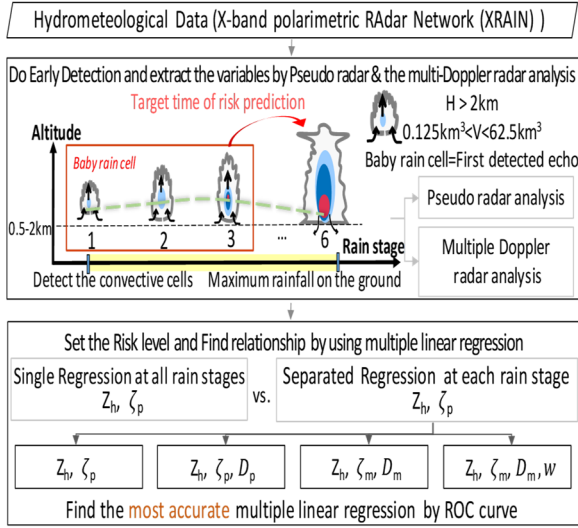


Fig. 3 Flowchart of improvement of the quantitative risk prediction accuracy. Z_h : Reflectivity, ζ_p : Pseudo Vorticity, D_p : Pseudo Divergence, ζ_m : Vorticity of multiple Doppler radar analysis, D_m : Divergence of multiple Doppler radar analysis and w : updraft

2.4 Quantitative Risk Prediction

After we find out the existence of the first echo, the rain stages from 2 to 6 are marked according to the process of convective cloud development with a time interval of 5 minutes. To set the predicted risk level, the predicted risk categories (risk levels) are defined at the maximum rainfall intensity on the ground, such as Risk 1 (under 30mm/hr), Risk 2 (between 30mm/hr and 50mm/hr), Risk 3 (between 50mm/hr and 70mm/hr), and Risk 4 (over 70mm/hr). Finding the relationship among the predicted risk level and variables (i.e. the reflectivity, vorticity, divergence, and updraft) is important within 30 minutes (from rain stage 1 to 6) before the maximum rainfall intensity occurs on the ground. Then, for predicting the risk levels by using the observed and calculated variables, the correlation can be represented by using multilinear regressions. Fig. 3 illustrates the process of how to find the relationship and select the most accurate multilinear regression by the quantitative risk prediction method. The formulation of the multilinear regression becomes;

$$PRL_i = C_{0_i} + C_{1_i} Z_{h_i} + C_{2_i} \zeta_i + C_{3_i} w_i + C_{4_i} D_i, \quad (4)$$

where PRL_i is predicted risk level from rain stage i . Z_h, ζ, w, D are reflectivity, vorticity, updraft, and divergence respectively. $i \in [1, 6]$ is the number of rain stage which represents the development of a cumulonimbus cloud. The variables would be extracted depending on each rain stage (Fig. 4). There is a multilinear regression which is performed with the variables (i.e. the reflectivity and vorticity) for all rain stages at once. Moreover, the stage dependent regressions are also considered to improve the accuracy of risk prediction by including the variables (i.e. the divergence and updraft), because the variables have characteristics depending on each rain stage.

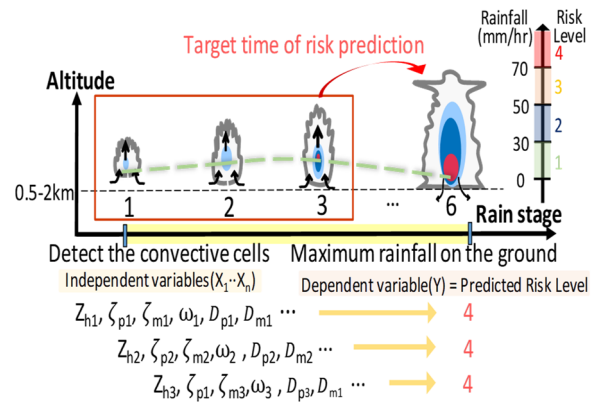


Fig. 4 The Early detection of the isolated convective cell until the maximum rainfall occurs on the ground. Extraction of variables depending on each rain stage.

To check the accuracy between total stage and stage dependent regressions, we apply the Receiver Operating Characteristic (ROC) analysis and examine the Area Under ROC Curve (AUC). The ROC analysis is a quantitative evaluation method to assess the accuracy of the multilinear regression. It divides values into observation and prediction as a contingency (Table 1). Also, it can estimate the ability of how many percent the analysis predicted the occurrence of GHR. The ROC curve plots the hit rate (HR) against the false alarm rate (FAR), which are computed as formulas (5) and (6):

$$\text{Hit rate (HR)} = H/(H+M), \quad (5)$$

$$\text{False alarm rate (FAR)} = F/(F+N), \quad (6)$$

where H and M represent hits and misses when GHR occurs. F and N represent false and negative hits when GHR not occurs. The ROC curve is plotted with the resulting pairs of (FAR, HR) from the contingency table are plotted and connected by line segments from the origin point (0, 0), which corresponds to never prediction of the heavy rainfall, and to the point (1, 1), which corresponds to always prediction of the heavy rainfall. The AUC can represent the accuracy of the multiple linear regression formulas with a range of 0.5 to 1. When the AUC value approaches unity, the ROC curve goes up to the upper-left corner of the ROC diagram and reflects better discrimination performance. Therefore, the most appropriate multiple linear regression formula is chosen by the highest AUC value among the identified multiple linear regression formulas. We analyzed the accuracy at the early and late rain stages as well to reveal specific characteristics of variables during each rain stage.

Table 1 ROC contingency table for RP_{RD} (Risk Prediction of radar) and RP_{RG} (Risk Prediction of regression).

		Observed Value (RP_{RD})	
		GHR occurred	GHR not occurred
Predicted Value (RP_{RG})	GHR occurred	Hit (H)	False (F)
	GHR not occurred	Missing (M)	Negative hit (N)

3. Data and Study Area

To provide the high spatiotemporal observation data throughout Japan, MLIT has been operating the X-band polarimetric RADar Network (XRAIN) since 2010. As of now, the MLIT has installed 39 X-band multi-parameter (X-MP) radars. Fig. 5 illustrates the positions of the radar sites and their observation ranges in the Kinki region. In our research target area, there are four radars which are named Rokko, Katsuragi, Juubusan, and Tanokuchi. The XRAIN in the Kinki region produces composite radar images every one minute

and provides complete volume scan data every 5 minutes. The three-dimensional volume scan data has a resolution of 250 and 500 m in the horizontal and vertical directions, respectively. Also, to retrieve the three-dimensional wind field, especially at upper levels, background information (i.e. Sounding data) is required. By using these data, the variables (i.e. the vorticity, divergence, and updraft) were calculated. From August 2013 to August 2018, 10 GHR events were selected and tabulated in Table 2.

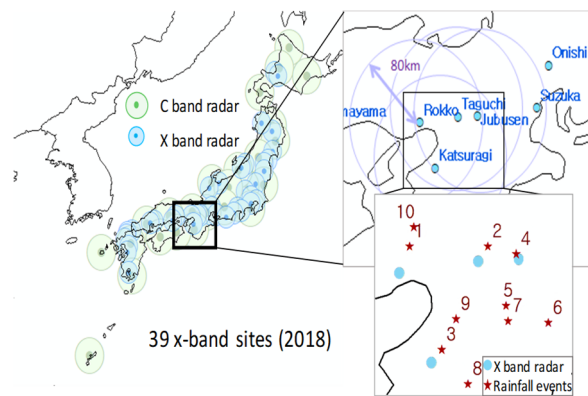


Fig. 5 The detection range of radars, study area, and distribution of Guerrilla heavy rainfall events.

Table 2 List of Guerrilla heavy rainfall events

No.	Date	No.	Date
1	2013-08-06	6	2015-08-29
2	2013-08-07	7	2016-08-03
3	2014-08-23	8	2016-08-25
4	2014-08-23	9	2017-08-04
5	2015-08-07	10	2018-08-13

4. Result and Discussion

4.1 Accuracy comparison between total stage regression and stage dependent regressions

To improve the quantitative risk prediction method, it is necessary to make regressions depending on each rain stage because the reflectivity and vorticity hold different characteristics in different development rain stages of a cloud. First of all, the performance of stage dependent regressions is compared with the total stage regression as a three-dimension scatter and

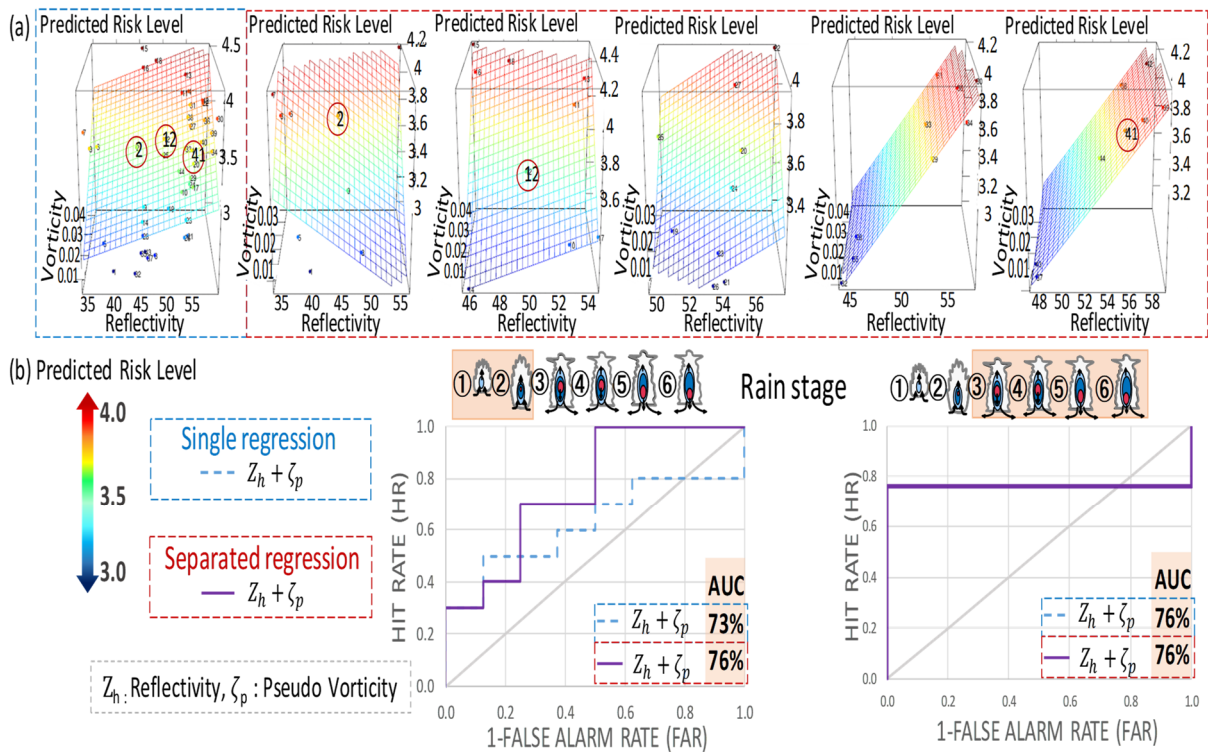


Fig.6 a) The performance of stage dependent regressions is compared with the total stage regression by three-dimension scatter and plane plot. b) The Comparison between total stage regression and stage dependent regressions is conducted based on the accuracy by ROC curve.

plane plot (Fig. 6a). The plane plots are useful for showing the relationship of regression analysis among the dependent variable (i.e. predicted risk level) and two independent variables (i.e. reflectivity and vorticity). The warmer color of the plane plot, which is closer to the risk level 4.0, corresponds to more severe risk levels. The total stage regression (Fig. 6a, blue dashed box) shows a positive correlation between the dependent variable and each independent variable. The stage dependent regressions (Fig. 6a, red dashed box) from rain stages 1 to 6 represent the relationship among the dependent variable and two independent variables. The correlation between predicted risk level and reflectivity shows more positive correlations as the rain stage develops, except rain stage 1. The correlation between predicted risk level and vorticity shows positive correlations at the early rain stage. However, at the late rain stage, it becomes difficult to find correlations. For each event, there are cases with rain stages 1 to 6. When each case is compared, the predicted risk level of stage dependent regressions shows higher than the total stage regression mostly.

Then, the accuracy is evaluated by using the ROC and AUC analysis. Fig. 6b represents the ROC curves at the early (i.e. rain stage 1 to 2) and late rain stages (i.e. rain stage 3 to 6), which is estimated by the total stage and stage dependent regressions with vorticity and reflectivity. At the early rain stage, the accuracy of stage dependent regressions is 76% (AUC=0.76), which is higher than the total stage regression (AUC=0.73). Because the number of cases about stage dependent regressions is smaller than the total stage regression statistically, the stage dependent regressions deviate from the limited number of cases. The stage dependent regressions are strongly affected by cases, so they have less statistical stability than total stage regression relatively. However, if we collect more events, the problem could be solved. Physically, the variables show different characteristics according to the development of the cloud. Also, the stage dependent regressions calculate higher accuracy at the early rain stage. For saving lives by alerting the warning at the beginning of GHR, this research is focused on stage dependent regressions.

4.2 Determination of the most appropriate regression among the variables

The development processes of the convective cloud at the rain stage 1, 2, and 5 are plotted in Fig. 7 with the variables calculated by Pseudo and multiple Doppler radar analysis. Depending on each rain stage, the three-dimensional vorticity and divergence are calculated. Then, the pair of vorticity, divergence, and convergence increase while the rain-cell developed into heavy rainfall. The divergence and convergence can be an explanatory variable because at the beginning of the cumulonimbus cloud the strong convergence occurred at the lower layer (Houze, 1997). In addition, when the vorticity is detected in the development of cumulonimbus cloud, a vertical vortex tube is formed by the strong updraft, and it brings the water vapor. According to the results, we consider that the vorticity, divergence, and updraft are related to the predicted risk level of heavy rainfall.

To analyze the characteristics of variables depending on each rain stage, the multilinear regressions are expressed by combining the divergence and updraft based on reflectivity and vorticity. In this paper, we use the part correlation coefficient (Huber, 1981) of variables because the part correlation coefficient shows how the explanatory variables explained the predicted risk level. A partial correlation coefficient shows a similar explanation with the part correlation coefficient. The differences are presented in the Appendix. The part correlation coefficient indicates that only the variable (i.e. vorticity) by eliminating the influence of the other variables can explain the dependent variable (i.e. predicted risk level). Fig. 8 illustrates the part correlation coefficient of variables affecting the predicted risk level depending on each rain stage by Pseudo and multiple Doppler radar analysis. It represents that the most explanatory variable is changed depending on each rain stage. In Pseudo and multiple Doppler radar analysis, the combinations of variables have conditions in common that the reflectivity and vorticity are very important at the early and late rain stage respectively. We focus on the early rain stage for saving evacuation time to

escape from danger because the Guerrilla heavy rainfall occurs within a few minutes. So, vorticity is the most explanatory variable to estimate the predicted risk level before the maximum rainfall reached the ground. Moreover, the inclusion of divergence and updraft can better describe the predicted risk level. By the way, the part correlation coefficient of updraft suddenly decreases at rain stage 3. The reason is that a transition occurs in which the direction of the updraft is changed to the downdraft.

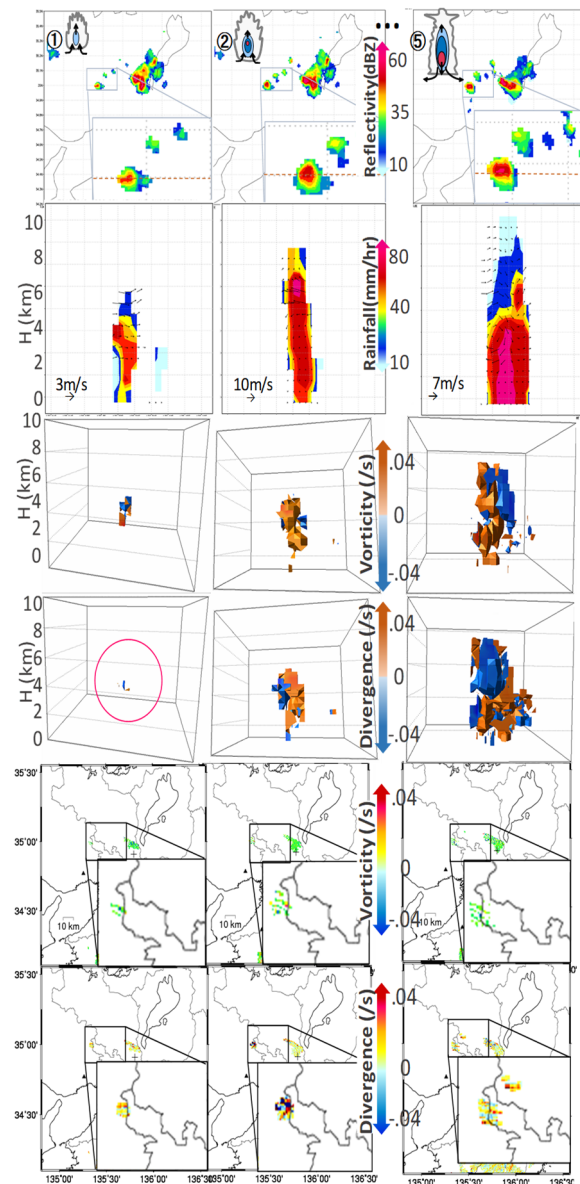


Fig.7 The reflectivity, updraft, vorticity, and divergence calculated by Pseudo and multiple Doppler radar analysis. The circle represents the existence of divergence and convergence at rain stage 1.

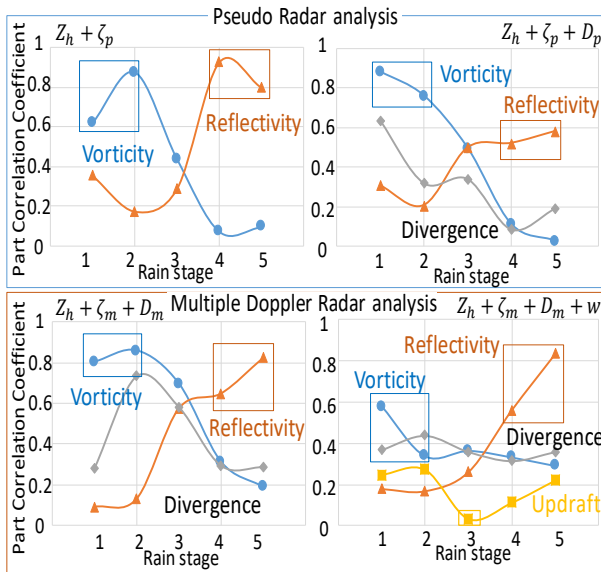


Fig.8 The impact of variables affecting the predicted risk level by Pseudo and multiple Doppler radar analysis.

The ROC analysis is applied to select the best multilinear regression with the highest accuracy. The predicted risk level is defined at the maximum rainfall intensity on the ground according to the predicted risk categories. To make the ROC curve, the HR, FAR, and AUC are estimated among the combinations of independent variables of the multilinear regression. Fig. 9 represents the ROC curves of the stage dependent regressions. As a result of the highest accuracy, the multilinear regression of reflectivity, vorticity, divergence, and updraft has 82% accuracy (AUC= 0.82). The regression by using multiple Doppler radar analysis and adding more independent variables has brought remarkable improvement rather than the total stage regression. Moreover, the inclusion of divergence and updraft can better describe the predicted risk level. The AUC analysis is conducted at the early and late rain stages and confirms higher accuracy at the early rain stage (90% accuracy). Therefore, by using the improved regression, the risk level of localized heavy rainfall could be predicted with high accuracy.

5. Conclusions

In order to minimize human injury such as isolation, death, and disappearance due to heavy

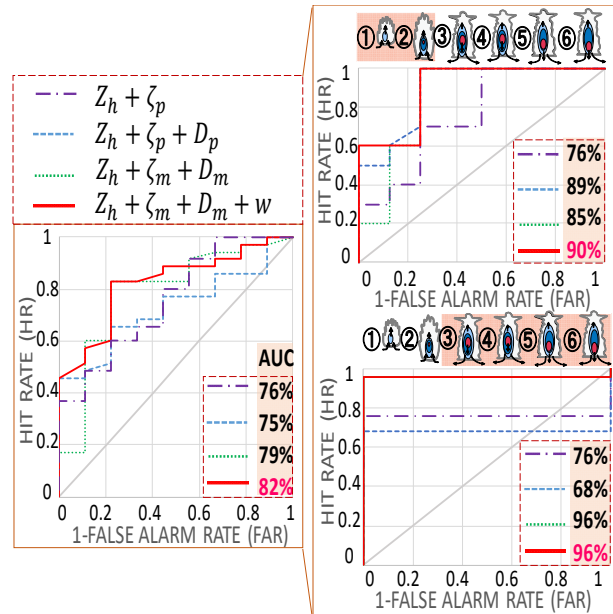


Fig.9 The accuracy of multilinear regression among the combination of dependent variables by AUC. By Pseudo radar analysis, the purple and blue lines were the regressions of 1) reflectivity, pseudo vorticity and 2) those with added pseudo divergence. By multiple Doppler radar analysis, the green and red lines were the regressions of 3) reflectivity, vorticity, divergence and 4) those with the added updraft.

rainfall, this study has proposed the quantitative risk prediction method which accurately alerts the risk of localized heavy rainfall. First, the stage dependent regressions depending on each rain stage are compared with the total stage regression by the performance of variables and accuracy since the reflectivity and vorticity show different characteristics. Then, by using Pseudo radar analysis and multiple Doppler radar analysis, the multilinear regression with the highest accuracy is chosen among the combinations of variables. If this regression is applied to the field, it is possible to secure more evacuation time and predict the disasters with high accuracy.

However, there still exists some points to improve. When we formulate the stage dependent regressions, the number of cases is less than the total stage regression. This condition could cause the stage dependent regressions to be over-fitted due to data insufficiency. If more cases are included, this problem should be resolved. Furthermore, it is still unclear to determine which

convective cell would be developed. Even if a baby rain-cell can be detected, it is difficult to predict in which direction the rain-cell would move and where heavy rainfall would occur. So, it necessitates finding more explanatory variables that affect the localized heavy rainfall to improve reliability and accurate risk prediction, such as reflectivity, rainfall intensity, Doppler wind speed, vorticity, the velocity of vertical invigoration, echo amplitude difference, convergence, etc. In addition, it is expected to carry out a study to predict the movement of convective cells by extrapolation to estimate the risk in long-term lead times.

The improved quantitative risk prediction (meteorology) and flash flood warning (hydrology) will be combined in the next step of this research. Flash floods happened when meteorological and hydrological circumstances coexist. Therefore, future research is promising to bridge the gap between this study and the flash flood prediction system.

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Appendix A. Differences between part and partial coefficient

The partial correlation coefficient shows a similar explanation with the part correlation coefficient. However, in this paper, we use the part correlation coefficient of variables because the part correlation coefficient shows how the explanatory variables explained the predicted risk level.

- ✓ The part correlation (Fig.S1) is the correlation between independent variable (X_1) and the dependent variable (Y) after the linear effects

of the other independent variables (X_2) have been removed from the independent variable (X_1) only. It can be calculated by the following formula.

$$r^2_{y,(x_1|x_2)} = B/(A+B+C+D) = B \quad (S.1)$$

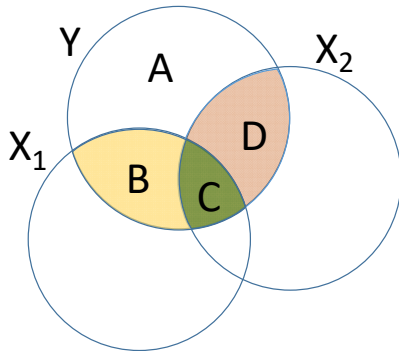


Fig.S1 The part correlation coefficient

- ✓ The partial correlation (Fig.S2) is the correlation between independent variable (X_1) and the dependent variable (Y) after the linear effects of the other variables (X_2) have been

removed from both the independent variable (X_1) and the dependent variable (Y). For example, there are the dependent variable (Y) and independent variables (X_1, X_2). The part correlation coefficient indicates the amount of unique explanatory power in Y explained by X_1 . It can be calculated by the following formula.

$$r^2_{(y,x_1)|x_2} = B/(A+B) \quad (S.2)$$

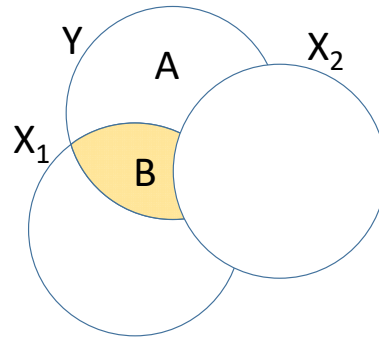


Fig.S2 The partial correlation coefficient

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