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Study on Applicability of Overflow Discharge Equation Under Pressurized Flow Condition

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Synopsis

Recently, urban inundation disasters resulting from torrential rain have led to serious problems in many countries, especially in Asian countries. The numerous cities have been inundated by storm water, which is the major cause of urban flooding. To mitigate the damage of urban inundation, an underground storage box has been implemented. However, there are no criteria on the degree of mitigation effect that can be expected from such underground storage systems. In order to evaluate the mitigation effect, appropriate discharge must be diverted from a sewerage system over the side weir. Hence, during this study, the applicability of De Marchi's equation in case of pressurized flow condition was validated using laboratory experiment results and suitable discharge coefficient was obtained for varying hydraulic conditions like weir length, weir height, etc. Finally, the numerical model was developed using one dimensional continuity and momentum equations with De Marchi's equation as overflow equation. The numerical results show good agreement with the experimental results and empirical correlation for coefficient of discharge is proposed for the side weir.

Keywords: Underground storage box, De Marchi's equation, Side weir, Discharge coefficient

1. Introduction

Even though numerous studies have been conducted on reducing urban inundation damage, such damage remains a serious problem every year. Urban inundation has caused immense property damage and many personal injuries due to short-term, local heavy rainfall and extreme climate conditions worldwide. Although wide, permeable areas such as farming land and forest may exist before urbanization, permeable areas decrease after urbanization as a result of increased road areas, building construction, etc. Due to this, most storm water does not infiltrate into the ground and the total amount of storm water runoff increases, which can lead to urban inundation.

To mitigate this problem, underground storage systems have been implemented as an effective countermeasure, especially in highly urbanized areas. However, there are no criteria on the degree of mitigation that can be expected from installing such underground storage systems. In many cases, these storage systems are attached to sewerage systems, and some portion of the storm water within a sewerage pipe is diverted over the side weir into the storage system. Therefore, an evaluation of storage systems' mitigation effects requires an appropriate estimation of overflow discharge from a sewerage system over the side weir. In this regard, it is essential to study the overflow discharge over the side weir related to underground storage systems. Fig. 1 briefly shows the structure of an underground storage system.

The present study was undertaken to determine the appropriate equation to estimate the diversion discharge to a storage box. De Marchi (1934) is the main contributor to the understanding of hydraulic behavior at side weirs. He presented a theory based on the assumption of a constant energy head along the side weir and the overflow discharge being calculated by the classical weir formula, which overlooks the effect of lateral outflow direction, local velocity, and type of flow (pressurized or non-

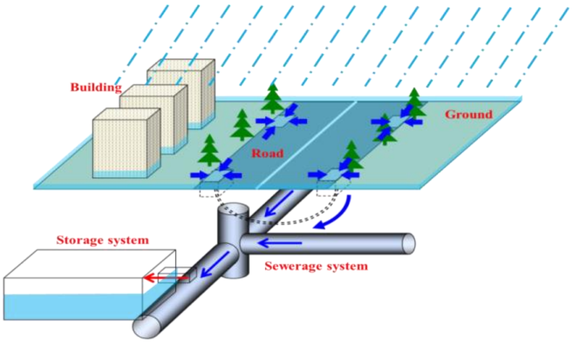

Fig. 1 Underground storage system

pressurized) in the system.

De Marchi's equation is usually employed in cases of open channel flow, and many researchers have suggested its discharge coefficient. However, no study has verified the suitability of De Marchi's equation in the pressurized flow condition. When considering overflow from a sewerage system during urban flooding, the pressurized flow condition would occur often and the applicability of De Marchi's equation must also be verified in that condition. Hence, in this study, an experimental set-up is proposed, in order to determine the discharge coefficient for pressurized flow in a circular channel with different side weir conditions. Such experimental data is then expected to be used to validate numerical models for estimating the effects of underground storage box.

2. Experimental Setup

The experiments were conducted at the Ujigawa Open Laboratory of the Disaster Prevention Research Institute (DPRI), Kyoto University, as shown in Fig. 2.

Fig. 2 Experimental setup

A rectangular side weir in a circular pipe is shown in Fig. 3, where D is the diameter of the main pipe, p is the height of the side weir, and L is the length of the side weir. The experimental setup consists of a side weir with two circular acrylic pipes that are 4m in length and 0.05m in internal diameter. An upstream supply tank with a recirculation pump system is present, as is a downstream collecting tank with a movable gate to adjust the downstream water level. The recirculation system can be controlled by an RPM controller, which controls motor speed to supply constant inflow discharge to the upstream tank. A flowmeter was used to measure the upstream input discharge. All of the experiments were conducted using a horizontal pipe. The experiments were conducted with three different side weir lengths, 10cm, 15cm, and 20cm, and three different side weir heights, 3cm, 3.5cm, and 4cm in all cases. The standard weir elevation is the dashed line on the bottom of the pipe in Fig. 3 and 4. This side weir height can be adjusted up and down easily by controlling the bolt. The fundamental length and height of the side weir model were determined according to the actual size of the pipe diameter and the side weir in Moriguchi City, which can be regarded as a typical overflow system of side weirs. Table 1 shows the ratios between the proto-type and the physical model's scale based on the similarity law.

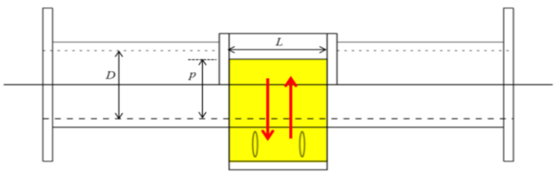

Fig. 3 Definition sketch of rectangular side weir

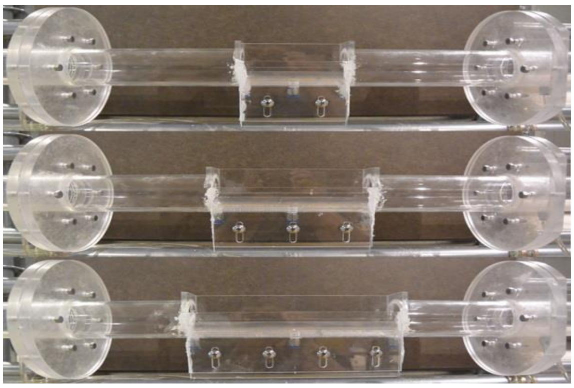

Fig. 4 Side weir

Table 1 Ratios between the proto-type and physical model's scale

Index	Ratio
Diameter	1/22
Weir length	1/23 ~ 1/45
Weir height	1/24 ~ 1/32

3. Experimental Conditions

63 experimental cases in total were conducted to determine the overflow discharge coefficient, with steady conditions but different side weir lengths and heights. The water heads were measured by a total of 17 piezometer tubes placed at the lowest bottom of the pipe. In particular, there are 3 measuring points along the bottom of the side weir section. Seven experiments were carried out for each different side weir condition, and the discharge supplied to the upstream tank differed from 0.5 ℓ/s to 1.1 ℓ/s. The downstream movable gate level was set to the bottom of the main pipe. The detailed hydraulic conditions are summarized in Tables 2, which contain the observed water head at the downstream end of the pipe. Each experiment under the same conditions was repeated three times to consider the consistency of the overflow discharge rate.

4. Experimental Results

Table 2 Experimental conditions

Weir length (cm)	Weir height (cm)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
Upstream discharge (ℓ/s)								
10, 15, 20	3, 3.5, 4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
Water depth at the downstream end of the pipe (cm)								
10	3	2.8	2.9	3.0	3.2	3.3	3.4	3.5
	3.5	2.9	3.0	3.0	3.2	3.3	3.4	3.5
	4	3.1	3.2	3.3	3.3	3.4	3.5	3.6
15	3	2.6	2.7	2.8	3.0	3.2	3.3	3.4
	3.5	2.9	3.0	3.0	3.2	3.2	3.3	3.4
	4	3.1	3.2	3.3	3.3	3.4	3.5	3.6
20	3	2.6	2.7	2.8	3.0	3.2	3.2	3.3
	3.5	2.8	2.9	3.0	3.1	3.2	3.3	3.4
	4	3.1	3.2	3.3	3.3	3.4	3.4	3.5

4.1 Water head profile

Fig. 5 shows some of water surface profiles around the side weir for the cases with weir lengths of 10, 15, and 20cm in weir height of 4cm. As can be seen in all of the figures, the water head at the upstream end of the side weir was lower than that at the downstream end of the side weir. This phenomenon was due to the water spurting quickly from the upstream to downstream portion, so that it takes the pressure at the downstream portion. As the upstream discharge increased, the hydraulic gradient became steeper, compared with the results of the smaller discharge cases. However, the variation of the water heads at the upstream end of the side weir was small, compared with the water head at the downstream end of the side weir for all of the cases. Due to this, if the water head at the upstream end of the side weir was used to calculate the discharge coefficient, the values increased in the greater discharge cases, with unstable values.

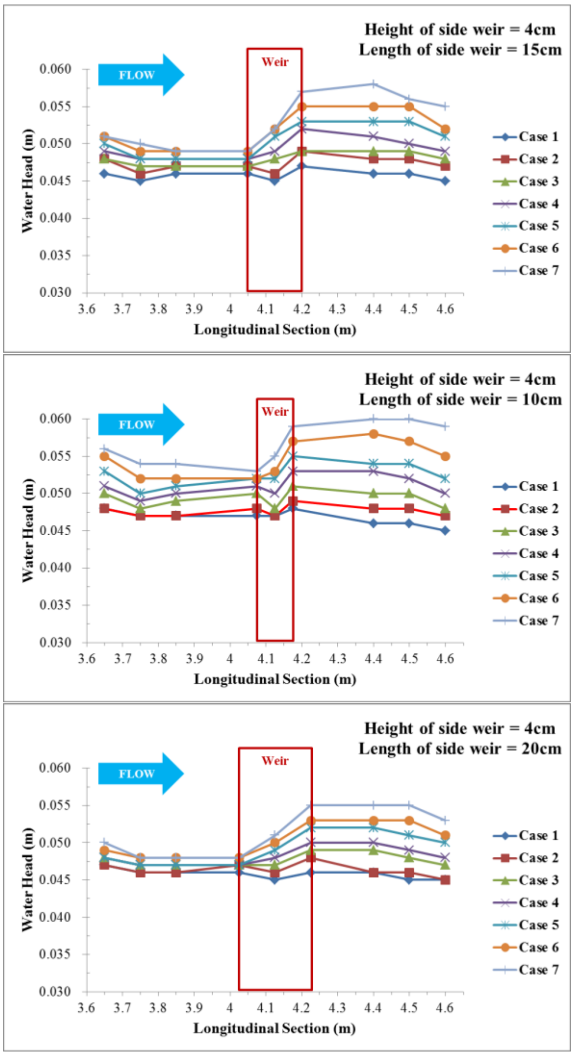

Fig. 5 Water surface profiles

The rate of the rise in pressurized flow conditions was larger than that in the open channel condition, and a high water head at the downstream end of the side weir has an effect on the water heads of the downstream pipe. The reason for this may be the strength of the lateral flow under pressurized flow conditions. This result shows the difference between open channel flow and pressurized flow conditions (Ko et al., 2014).

4.2 Discharge coefficient

De Marchi's equation was adopted to calculate the overflow discharge in this model, which is as follows (1):

$$C_d = \frac{\frac{3}{2}Q_{out}}{\sqrt{2g}(h-p)^{\frac{3}{2}}} \tag{1}$$

This discharge coefficient depends on the hydraulic conditions including the weir length, weir height, and water head on the side weir and the overflow discharge. As the upstream discharge increases, the overflow rate also increases, in comparison with the results under the low-discharge conditions. Table 3 shows the overflow discharge along each weir condition, which increases in accordance with the lower height and longer length of the side weir. The seven discharge coefficients were derived using the above hydraulic conditions for each different weir condition. Some previous studies used the water head at the upstream end of the side weir to calculate the discharge coefficient. However, as I have already mentioned above, there is excessively larger variation in the water heads on the side weir, compared with those of previous studies on the

Table 3 Experimental results (overflow discharge)

Weir length (cm)	Weir height (cm)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
Experiment-overflow discharge (ℓ/s)								
10	3	0.132	0.194	0.238	0.304	0.344	0.418	0.456
	3.5	0.094	0.154	0.204	0.268	0.320	0.380	0.424
	4	0.063	0.113	0.168	0.233	0.279	0.324	0.366
15	3	0.136	0.224	0.264	0.320	0.384	0.426	0.504
	3.5	0.112	0.168	0.236	0.290	0.350	0.404	0.466
	4	0.065	0.122	0.179	0.246	0.304	0.377	0.430
20	3	0.164	0.224	0.284	0.324	0.394	0.448	0.514
	3.5	0.118	0.174	0.240	0.300	0.360	0.414	0.480
	4	0.069	0.130	0.199	0.253	0.330	0.407	0.468

characteristics of pressurized flow. From this perspective, the mean of the integral water heads along the entire side weir length was adopted, which was derived from the side weir's length and the water head integrated along the side weir.

Uyumaz et al. (1985) also reported that the water head is not constant on a side weir. The mean of the upstream and downstream water heads on the side weir did not produce satisfactory solutions. Calculating the average of several intermediate heads provided more satisfactory results.

In this study, the coefficient value increased gradually when the water head at the upstream end of the side weir was used. In contrast, the discharge coefficient remained almost constant when the mean of the integral water heads was adopted, instead of the water head at the upstream end of the side weir. In particular, these coefficient values became close to the constant value, even though the upstream discharge increased, implying the applicability of a constant coefficient value. Fig. 6 shows one of cases with weir length of 10cm in weir height of 4cm for the discharge coefficient distribution along the different water head on the side weir.

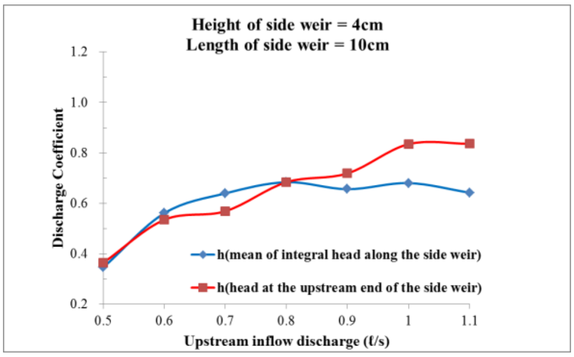

Fig. 6 Discharge coefficient distribution

5. Numerical Simulation Model

5.1 1-D sewer model

The storm water in the sewer pipe is dynamically calculated based on the following 1D continuity (2) and momentum (3) equations:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_{out} \quad (2)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial (uQ)}{\partial x} = -gA\frac{\partial H}{\partial x} - \frac{gn^2|Q|Q}{R^{\frac{4}{3}}A} \quad (3)$$

where A is the cross-sectional area of flow, Q is the discharge, q_{out} is the overflow discharge per unit length over the side weir, u is the flow velocity, n is the Manning's roughness coefficient, R is the hydraulic radius, H is the piezometric head ($H = z + h$), z is the bottom elevation of the pipe, and h is the water depth in the pipe measured from the bottom, which is calculated as follows (4):

$$h = \begin{cases} f(A) & : (A \leq A_p) \\ B\grave{} + \frac{A - A_p}{B_s} & : (A > A_p) \end{cases} \quad (4)$$

where f is a function that expresses a relationship between the flow cross-section area and water depth in a circular pipe, the case of $A \leq A_p$ is the open-channel flow condition and $A > A_p$ is the pressurized flow condition, $B\grave{}$ is the height of the pipe ceiling, A_p is the cross-sectional area of the pipe, and B_s is the slot width, which is calculated as follows (5):

$$B_s = \frac{gA}{a^2} \quad (5)$$

where a is the pressure propagation velocity for the pipe and 5 m/s was used. The value of the roughness coefficient is uniformly 0.013. In other to treat both open-channel and pressurized flow conditions, the slot model, which considers the sewer pipe with a hypothetical narrow slot on its ceiling, is introduced in this study (Chaudhry, 1979).

5.2 Overtopping equation

De Marchi's equation was adopted to calculate the overflow discharge in this model, which is as follows (6):

$$q_{out} = \frac{dQ_{out}}{dL} = \frac{2}{3}C_d\sqrt{2g}(h-p)^{1.5} \quad (6)$$

where q_{out} is the discharge per unit length of the side weir, Q_{out} is the overflow discharge, L is the distance along the side weir measured from the upstream end of the side weir, g is the acceleration of gravity, p is the height of the side weir, h is the flow depth at the section L, and C_d is the discharge coefficient of the side weir.

A rectangular mesh was used, and the mesh generated depends upon the length of the side weir used in the model. That is to say, the weir length was reflected by the grid size, and a finite different method was adopted.

6. Model Verification

Under the assumption that De Marchi's equation is applicable to the pressurized flow condition, the numerical simulation was carried out to verify the suitability of the discharge coefficient obtained from the experiment.

First, those discharge coefficients for each different weir condition were applied to numerical simulation. Secondly, the each overflow discharge simulated was compared with overflow discharge data obtained by experiment. Consequently, the overflow discharge simulated could reproduce the experimental results though there are still small different on the whole, as shown in Fig. 7. The small difference is caused by inevitable measuring error of sampling method. This reproduction implies the applicability of De Marchi's equation to pressurized flow.

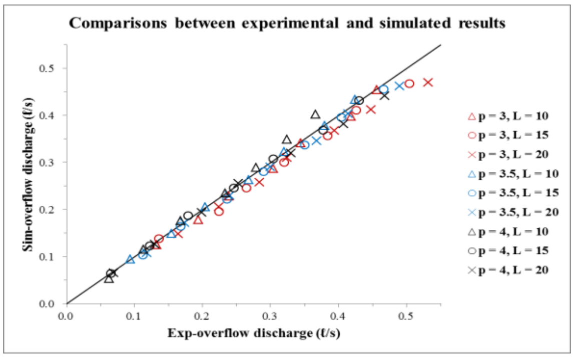

Fig. 7 Comparisons between the experimental and simulated results

When applying the overflow discharge coefficient to the actual field, the constant coefficient value would be easy to handle. Therefore, a suitable coefficient was determined within the limited experimental results for each different side weir condition. As a result, suitable coefficients from 0.57 to 0.64 were suggested. Additionally, an empirical correlation to predict the discharge coefficient was developed for the ratio of weir height to length using the suitable coefficients. However, previous C_d empirical correlations in terms of the above function are not applicable to my study because each experiment was carried out along different experimental conditions, like open channel or differing weir scale. The applicability of *h* and *F* on the equation of C_d is limited due to the limited input discharge range in the experiment, compared with other research. Thus, in this study, I decided to use the function of *p/L*, since p and L are already fixed, as shown in Fig. 8. The resulting correlation is given in Equation (7):

$$C_d = -0.2604\frac{p}{L} + 0.6736 \tag{7}$$

where the deterministic coefficient(R^2) is 0.9619.

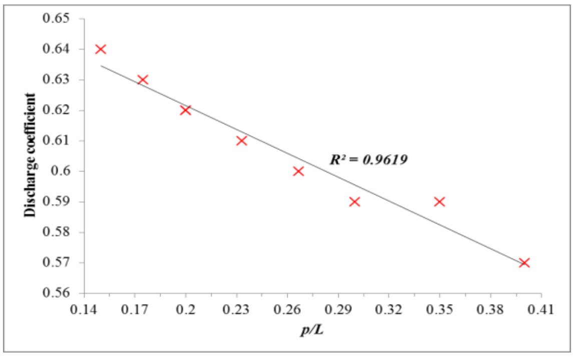

Fig. 8 C_d empirical correlations using the function of *p/L*

Suitable values were selected within the range of the lowest and highest coefficients in the experimental results along the weir condition and were determined through the least square method, where the triangle is the lowest coefficient, the circle is the highest coefficient, and the cross is the most suitable coefficient, as shown in Fig. 9.

7. Summary

In this study, under the assumption that De Marchi's equation is applicable to the pressurized flow condition, the numerical simulation was carried out to verify the suitability of the discharge coefficient obtained from the experiment. First, the discharge coefficients for each different weir condition were applied to numerical simulation. Secondly, each overflow discharge simulated was compared with the overflow discharge data obtained from the experiment. Consequently, the overflow discharge simulated could reproduce the experimental results, although there are still small differences as a whole.

When applying the overflow discharge coefficient to the actual field, the constant coefficient value would be easy to handle. Therefore, a suitable coefficient was determined

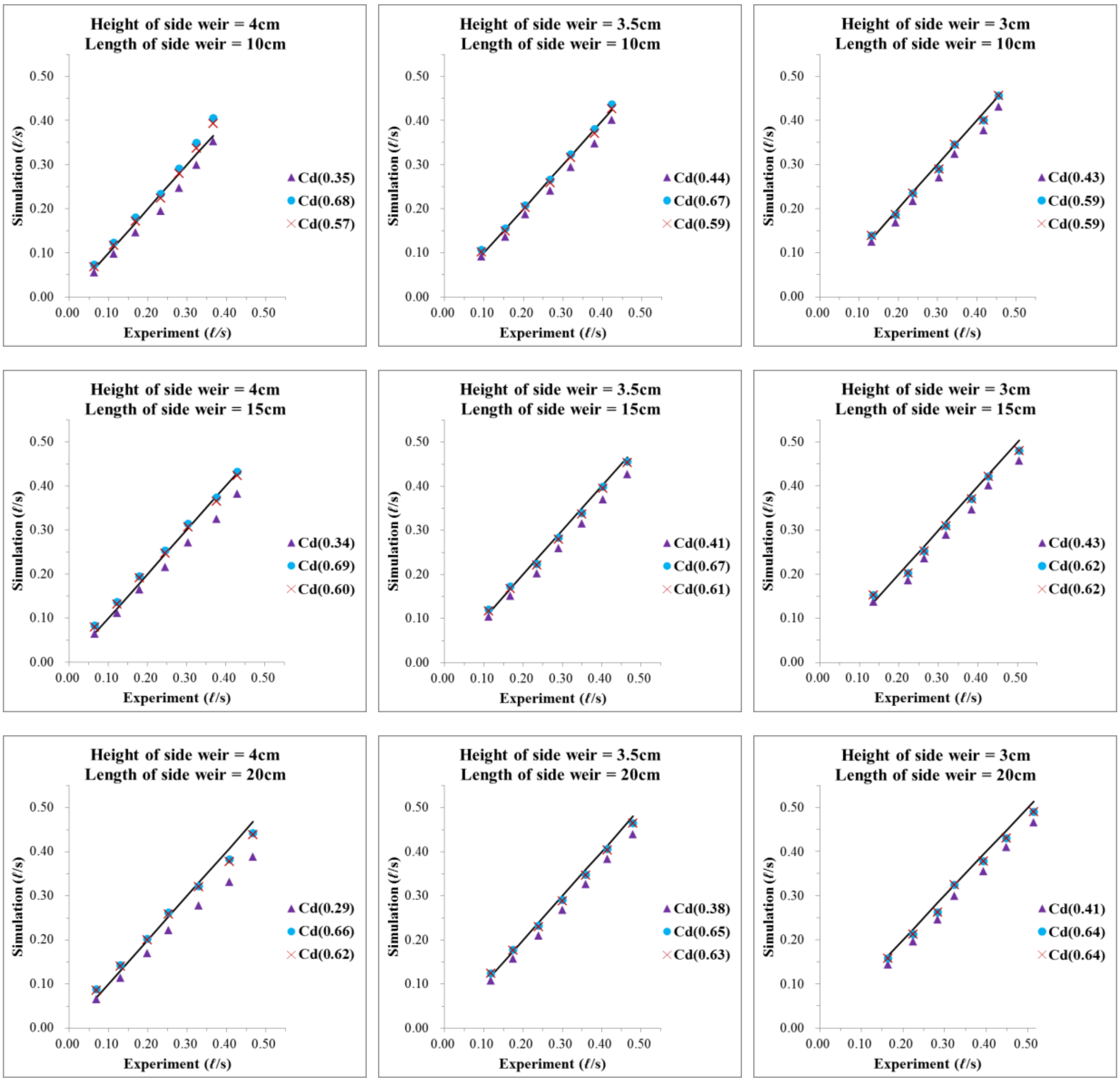

Fig. 9 Suitable discharge coefficients under different weir conditions

within the limited experimental results for each different side weir condition. As a result, suitable coefficients from 0.57 to 0.64 were suggested. Additionally, an empirical correlation to predict the discharge coefficient was developed for the ratio of weir height to length using the suitable coefficients.

However this study is limited to the steady condition only for simplicity. Next step will be to verify a numerical model under unsteady condition, which would enable to estimate the overflow discharge from sewerage to storage box and the mitigation effects of these storage boxes.

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References

Chaudhry, M. H.(1979): Applied Hydraulic Transients, Van Nostrand Reinold.

De Marchi, G.(1934): Saggio di teoria del funzionamento degli stramazzi laterali, L'Energia Elettrica, Vol.11, No.11, pp. 849–860 (in Italian).

Ko, D., Nakagawa, H., Kawaike, K. and Zhang, H.(2014): Study on estimation of inflow discharge to underground storage system for mitigation of urban inundation damage, J. Japan Society for Natural Disaster Science, Vol. 33, pp.

177-184.
Uyumaz, A. and Muslu, Y.(1985): Flow over side weirs in circular channels, J. Hydraulic Eng., Vol. 111, No.1, pp. 144-160.

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