

Density Current Simulation Using Two-Dimensional High Resolution Model

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Synopsis

A two-dimensional multi-phase numerical model for incompressible, immiscible and variable density fluids has been developed based on Navier-Stokes equations. The governing equations were discretized in a collocated grid system and solved with a finite volume method. The present model was parallelized in a shared memory system using openMp which reduced the computational time. The model was verified against Boussinesq and non-Boussinesq lock exchange problems. In addition, the model was applied to simulate salinity intrusion into Ohashi River that connects Nakaumi and Shinji brackish coastal lakes. The developed model successfully predicted the density current propagation in the experimental cases while it still needs refinement for field study cases.

Keywords: Numerical modeling, Multi-phase flow, Density current, OpenMP

1. Introduction

Gravity or density currents constitute a large class of natural flows that are generated and driven by the density difference between two or even more fluids. The density difference between two fluids usually arises due to differences in temperature or salinity, but it can also arise due to the presence of suspended solid particles. In water environments, saline intrusions or oil spills in the oceans, turbid water intrusions in a lake and suspended sediment plumes in a river are good examples. In case of shallow coastal lakes or estuaries, usually there is an interaction between sea water and the lake's fresh water. During tide, the sea water propagates into the lake, causing the sea water to plunge underneath the free surface of a receiving water basin. The plunge sea water forms a density current that continues to flow along the bottom causing water stratifica-

tion. This stratification acts as a barrier restraining the mixing of the water column in coastal lakes. As a result of the incomplete mixing of the water column and lack of light for the photosynthesis in deeper layers, the water column may become anoxic. Assessing the exchanges between water layers is important because they influence the trophic organization of the ecosystem. Density stratification in lakes, estuaries and in the oceans leads to circulation and mixing of large volumes of water resulting in multiphase flow. Modeling of these processes is of great importance for conservation of the ecological balance of these systems.

Several laboratory experiments have been performed to study density currents (e.g. Lowe et al., 2005; Kolar et al., 2009). With the rapid development of numerical methods and advancements in computer technology, Computational Fluid Dynamics has been widely used to study density currents. Mathematical and numeri-

cal models when properly designed and tested against field or laboratory data, can provide significant knowledge for density current dynamics and characteristics. The characteristics of a gravity current head have been studied by means of Large-Eddy Simulation (LES) and 3D Direct Numerical Simulations (DNS) of flow fronts in the lock-exchange configuration (e.g. Hartel et al., 2000, Ooi et al., 2007). Other researchers studied the density current using commercial CFD codes based on multiphase flow as FLUENT and FLOW-3D (e.g. Georgoulas et al., 2010; Sangdo et al., 2012). A number of hydrodynamic models are also used to simulate density currents in lakes, estuaries and coastal seas (e.g. Kasem and Imran, 2004; De Cesare et al., 2006; Ushijima et al., 2007)

In this paper, a 2D vertical multiphase model has been developed for studying salinity propagation process in brackish coastal lakes due to tidal water flux fluctuations. The present model describes a multiphase system for immiscible and incompressible fluids with collocated grid system which are parallelized in a shared memory system by Open Multiprocessing (OpenMp, 2011). The present research aims to present the validity, usefulness and applicability of multiphase numerical approach for the simulation and study of the hydrodynamic and salinity intrusion that are usually formed at estuaries and coastal lakes due to tide effect representing the water surface fluctuations as well as the density current propagation. The developed model is validated against the experimental data for Boussinesq and non-Boussinesq lock-exchange problems. In addition, saltwater intrusion into the tidal Ohashi River is simulated and validated using field measured data.

2. Numerical Procedures

2.1 Flow governing equations

The basic governing equations for flow are the mass conservation equation and momentum equations, Eq.(1)-Eq.(2), which describe the flow conditions in the vertical domain Ω as follows:

$$\int_{\Omega} \frac{\partial \rho}{\partial t} d\Omega + \oint_{\partial\Omega} \rho \mathbf{u} \cdot \mathbf{n} dl = 0 \quad (1)$$

$$\int_{\Omega} \frac{\partial u_i}{\partial t} d\Omega + \oint_{\partial\Omega} u_i \mathbf{u} \cdot \mathbf{n} dl = \int_{\Omega} f_i d\Omega - \frac{1}{\rho} \oint_{\partial\Omega} p \cdot \mathbf{n} dl + \oint_{\partial\Omega} \nu \nabla u_i \cdot \mathbf{n} dl \quad (2)$$

where t is time; ρ is the cell averaged fluid density; \mathbf{u} is the velocity vector; u_i is the velocity component with subscript $i = 1$ and 2 in x - and z - directions, respectively; x and z are the vertical orthogonal coordinates; \mathbf{n} is the unit normal vector on the domain boundary $\partial\Omega$ towards the outside direction; p and ν are the average fluid pressure and dynamic viscosity, respectively; and f_i is the external force component due to acceleration.

2.2 The interface tracking method

The motion of the interface between immiscible liquids of different densities and viscosities is defined by the volume fraction function ϕ and the interface is convected using the computed velocity field by the following three conditions:

$$\phi = \begin{cases} = 1, & \text{fluid 1} \\ 0 < \phi < 1, & \text{interface} \\ = 0, & \text{fluid 2} \end{cases} \quad (3)$$

The time dependence of the volume fraction function ϕ is governed by:

$$\int_{\Omega} \frac{\partial \phi}{\partial t} d\Omega + \oint_{\partial\Omega} \mathbf{u} \phi \cdot \mathbf{n} dl = 0 \quad (4)$$

Eq.(4) states that ϕ moves with the fluid and is considered as a flag identifying cells that contain fluid. According to the local value of ϕ , appropriate properties and variables are assigned to each cell within the domain.

2.3 Convection diffusion equation for transported concentration

The transported concentration is obtained by solving the convection-diffusion equation:

$$\int_{\Omega} \frac{\partial c}{\partial t} d\Omega + \oint_{\partial\Omega} \mathbf{u} c \cdot \mathbf{n} dl = \oint_{\partial\Omega} \frac{\nu}{\sigma_c} \nabla c \cdot \mathbf{n} dl \quad (5)$$

where c is the fluid concentration, and σ_c is Schmidt number for concentration diffusion, which represents the ratio of viscosity to fluid diffusivity.

2.4 Computational procedure

The Numerical computation consists of four stages: flow computation, interface tracking, mass transport, and updating fluid properties stages. The governing equations are discretized in a collocated grid system and solved with a finite volume method.

- (i) In flow computation stage; the flow equations, Eq.(1)-Eq.(2), are solved according to the existing domain and boundary conditions. Estimated velocity components are obtained at the cell center using pressure at the present computational step while upwind scheme is used to calculate convective terms. The cell center velocity components are interpolated at the cell boundaries. C-HSMAC algorithm is used to correct the pressure field so that the divergence of velocity in a cell should be less than a small threshold value (Ushijima et al., 2003). This algorithm for pressure computation has the power to deal with large density contrast fluids (Ushijima et al., 2007). The cell centered velocity components then are corrected with the obtained pressure values.
- (ii) In the interface tracking stage; the volume fraction function ϕ is calculated from Eq.(4) using the computed velocity field. The interface between different fluids is achieved and hence various fluid zones can be determined.
- (iii) In the mass transport stage; concentrations can be estimated using the convection-diffusion equation, Eq.(5), using the computed velocity field.
- (iv) In the updating fluid properties stage; a simplified equation of state is used to compute fluid density according to the volume fraction function and concentration values, given in a simple linear equations:

$$\rho = \phi\rho_l + (1 - \phi)\rho_g \quad (6)$$

$$v = \frac{\phi\rho_l v_l + (1 - \phi)\rho_g v_g}{\rho} \quad (7)$$

with

$$\rho_l = \rho_o + \beta(c - c_o) \quad (8)$$

where ρ is the cell average density according to the existing fluid; ρ_l and ρ_g are the densities of

two-phase immiscible fluids (liquid and gas); v is the cell average viscosity according to the existing fluid; v_l and v_g are the viscosities of the two-phase immiscible fluids; ρ_o and c_o are the reference density and concentration values which are usually taken as 0.998 g/cm^3 and zero, respectively, for fresh water; and β is the direct proportional coefficient of the salinity gradient between the maximum and minimum value.

- (v) Finally, the model goes to the flow computation stage again to calculate the water pressure and the flow velocity field and so on.

2.5 Open multiprocessing computation technique (OpenMP)

A parallel computation technique in shared memory system is implemented in the present model. This model is parallelized by Open Multiprocessing (Open MP, 2011). OpenMP is an implementation of multi-threading which is a method of parallelizing whereby a master thread (a series of instructions executed consecutively) forks a specified number of slave threads and a task is divided among them, Fig.1. The threads then run concurrently, with the FORTRAN runtime environment allocating threads to different processors. This process enables us to significantly reduce the elapsed time of the large-scale computations.

3. Model Validation With Experimental Results

The present model has been tested to evaluate its applicability. Therefore, it was applied to density current problems and compared with experimental results and field data. The simulation results for these test cases are described below.

3.1 Lock-exchange problem

The lock-exchange test or dam break problem is defined as water of different densities are separated by a vertical barrier which is removed at time zero. Lock exchange involving fluids either with small density differences (the Boussinesq case) or with large density differences (the non-Boussinesq case). The removal of the barrier results in a gravity current of the lighter

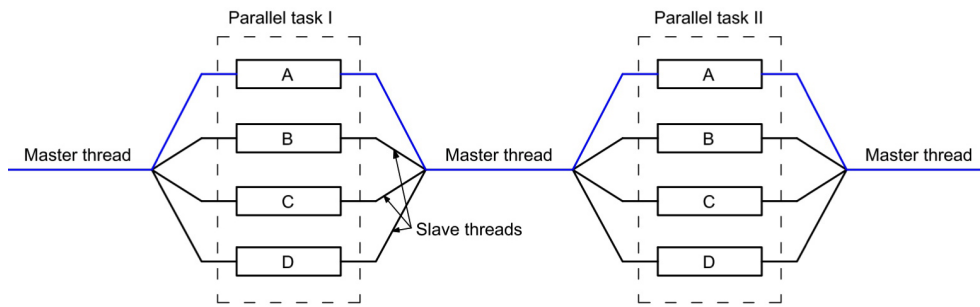


Fig .1 An illustration of multithreading process where the master thread forks off a number of threads which execute blocks of code in parallel.

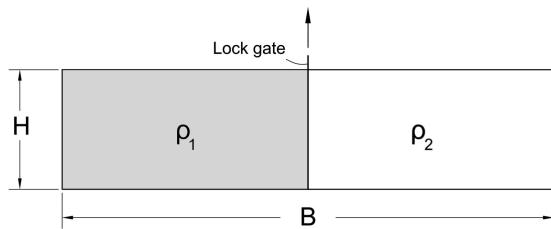


Fig .2 Sketch of the lock-release tank.

fluid propagating at constant speed along the upper surface of the channel into the heavy fluid. In the opposite direction a gravity current of the heavier fluid propagates also at constant speed along the bottom of the channel. The lock-exchange test represents mixing processes that occur frequently in geophysical fluid dynamics. For the Boussinesq case: when a fresh water river empties into a salt water estuary while for the non-Boussinesq case: the volcanic eruptions often take the form of gravity currents. The density within the flow is a result of suspended ash and hot rocks, and is often many times larger than the surrounding air. Fig.2 presents a sketch for the used the lock-release cell, showing two fluids of densities ρ_1 and ρ_2 , each having a depth of H and separated by a removable lock gate. Herein, the Boussinesq and Non-Boussinesq lock-exchange tests have been used to validate the developed code.

3.2 Boussinesq lock-exchange experiment

This model was applied to the Boussinesq lock-exchange experimental results obtained by Kolar et al. (2009). The density cell had internal dimensions of

length (B) = 58.4 cm, height (H) = 29.5 cm, and width = 2.54 cm. The both mixing fluids were initially separated by the barrier which was 0.4 mm thick of stainless steel sheet. The both mixing fluids were fresh water and saline water with a salt concentration of 17.5 ppt. Water at room temperature was used, resulted in a heavy water density (ρ_1) of = 1.011 g/cm³ and a light water density (ρ_2) of 0.998 g/cm³, for a density ratio of 0.987, which was clearly in the Boussinesq realm.

Herein, we present high resolution outcomes for direct and quantitative comparison of experimental and numerical results throughout the domain, not just the wave front. Fig.3 compares the model results with the laboratory experiments and determines how well the model captured the evolution of a density field that is driven entirely by gravity currents. This figure clearly shows a good agreement between the calculated density propagation of the wave front compared with those which are experimentally measured. In addition, a coincidence between the front position for the estimated and the measured data was obtained as shown in Fig.4.

3.3 Non-Boussinesq lock-exchange experiment

Lowe et al. (2005) have carried out a series of experiments on the lock exchange involving fluids with large density differences - the non-Boussinesq case. One of the experiments is selected in this study. The experiments were performed in a rectangular channel using two fluids of density ratio ($\gamma = 0.681$). The less dense fluid was freshwater ($\rho_2 = 0.998 \text{ g/cm}^3$) and the denser fluid was either a solution of sodium chloride (NaCl)

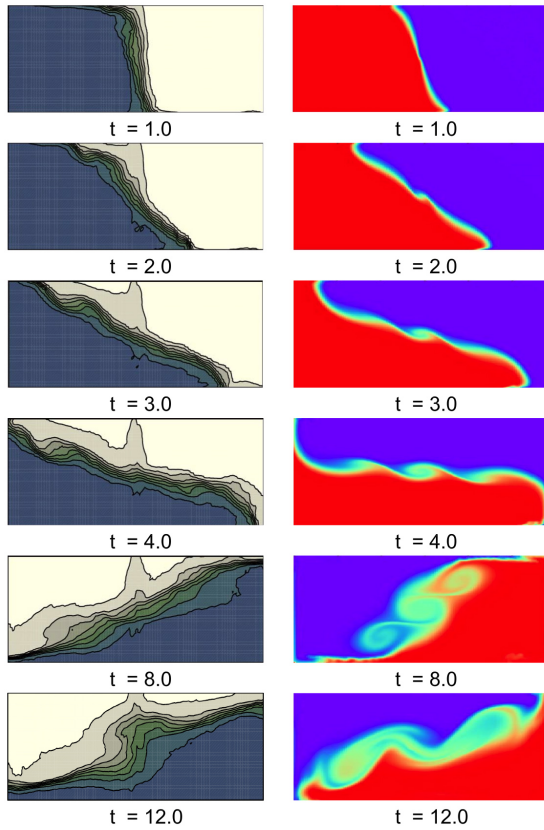


Fig .3 Comparison of experimental and predicted results for the density current propagation sequences for the Boussinesq lock-exchange case; (left) laboratory results and (right) model results.

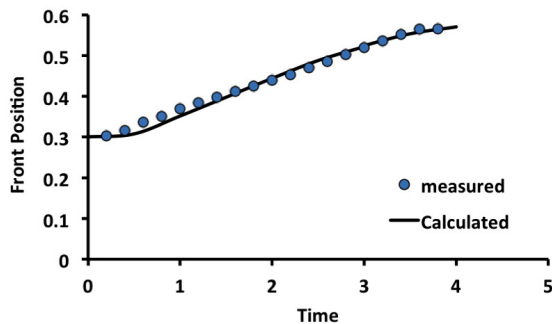


Fig .4 Comparison of the front position estimated from the laboratory experiments and the present model for the Boussinesq lock-exchange case.

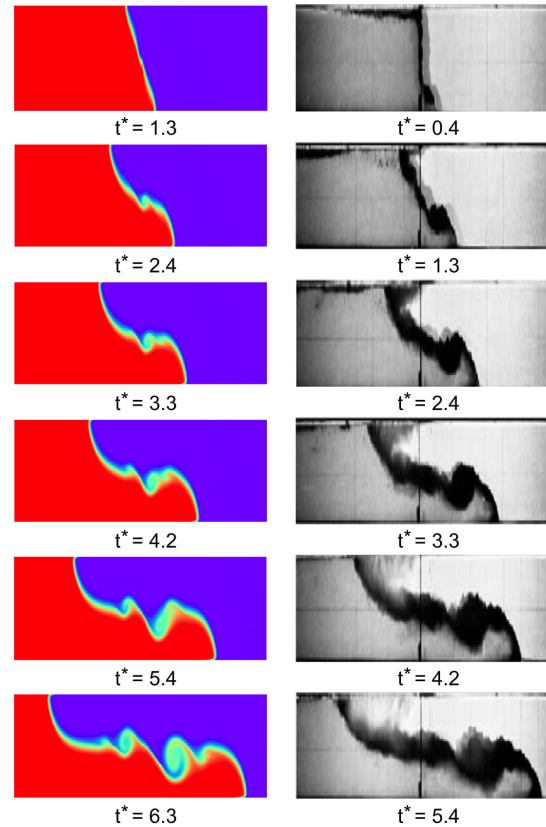


Fig .5 Comparison of experimental and predicted results for the density current propagation sequences for the non-Boussinesq lock-exchange case, (where $t^* = t \sqrt{(g(1 - \gamma))/H}$); (left) model results and (right) laboratory results.

or sodium iodide (NaI) ($\rho_1 = 1.466 \text{ g/cm}^3$). The channel length was $B = 182 \text{ cm}$, 23 cm wide and was filled to a depth of $H = 20 \text{ cm}$. The flow was started by a vertically removing lock gate.

Fig.5 compares the model results with the laboratory experiments and determines how well the model captured the evolution of a density field that is driven entirely by gravity currents. This figure clearly shows an acceptable agreement between the calculated density propagation of the wave front compared with those which are experimentally measured but in a different time scale. In addition, the developed model successfully predicted the bore within the heavy front. The front velocity of the density current is almost the same for the light case as well as the heavy case as shown in Fig.6. In the heavy front case, discrepancy may be

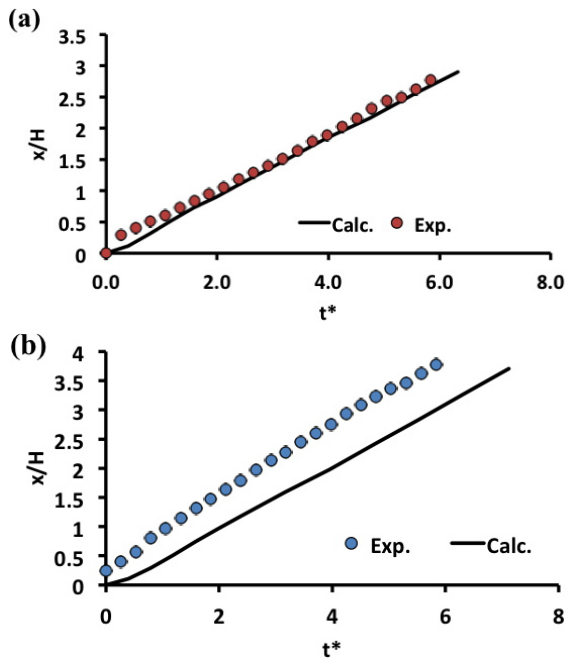


Fig. 6 Comparison of the relative front position estimated from the laboratory experiments and the present model for the non-Boussinesq lock-exchange case; (a) light front and (b) heavy front

observed because of the gate removal effect.

4. The Model Application on Ohashi River, Japan

The Ohashi River in southwest Japan is a tidal river located between two brackish lakes, Shinji and Nakaumi, Fig.7. These brackish lakes are stably stratified due to salinity (density) differences. Nakaumi Lake is strongly affected by the salt water entering from the Sea of Japan through Sakai channel while the freshwater Hii River influenced on Lake Shinji. The density gradient has a significant impact on water movement in the Ohashi River. In this study, the field measured data (September 10, 1999) for the saltwater intrusion into the Ohashi River are used to validate the present model (Moriwaki et al., 2003). Fig.7 shows the nine measuring stations along the Ohashi River.

The salinity initial conditions of the salinity concentration values, computational domain, and the two lakes water levels are introduced in Fig.8. The model run starting time was 10:30 Am. The used simulation time period was chosen to decrease the effect of tem-

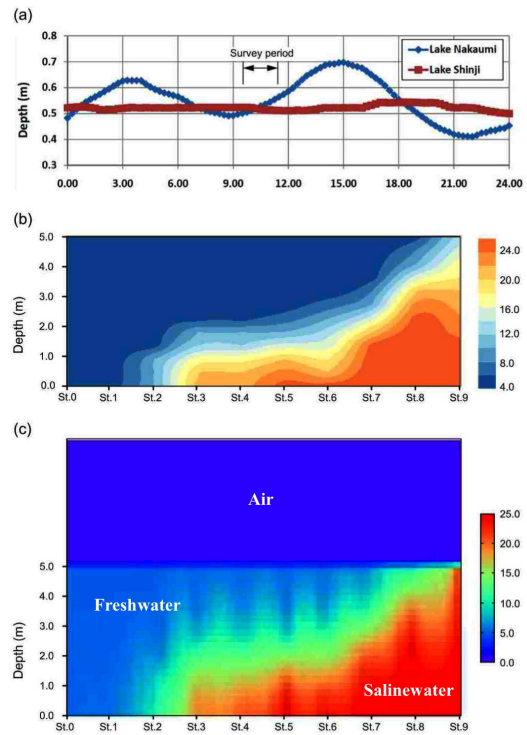


Fig. 8 Initial conditions at 10:30 Am: (a) water level fluctuation, (b) measured salinity values along the Ohashi River, and (c) estimated salinity values along the Ohashi River.

perature on density values. The computational domain was separated vertically into two phases, air at top and water at bottom with different concentrations. A computational time step of 0.1 sec. and a computational grid of 0.25m and 100m in the vertical and the horizontal directions, respectively, were used in the numerical calculations. The computations were performed using 32 threads which could reduce the computational time by 85% compared with the case of using single thread. The linear wave theory was used to calculate the intrusion water flux from Lake Nakaumi using the water level difference between the two lakes. The model results show good agreement against the measured data for salinity concentrations at 14:30 Pm and 16:30 Pm as shown in Fig.9 and Fig.10. However, model refinement is needed to avoid the stripping shape of salinity concentrations.

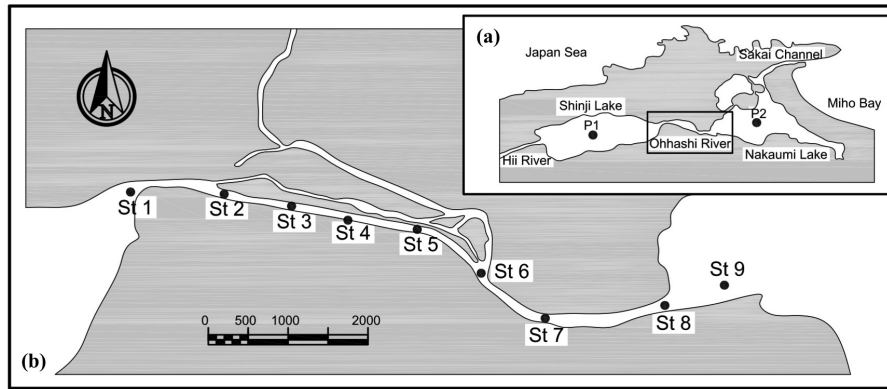


Fig. 7 Geographical location of the Ohashi River and sampling locations: (a) location in the Lake Shinji-Nakaumi system and (b) nine sampling localities in September, 1999.

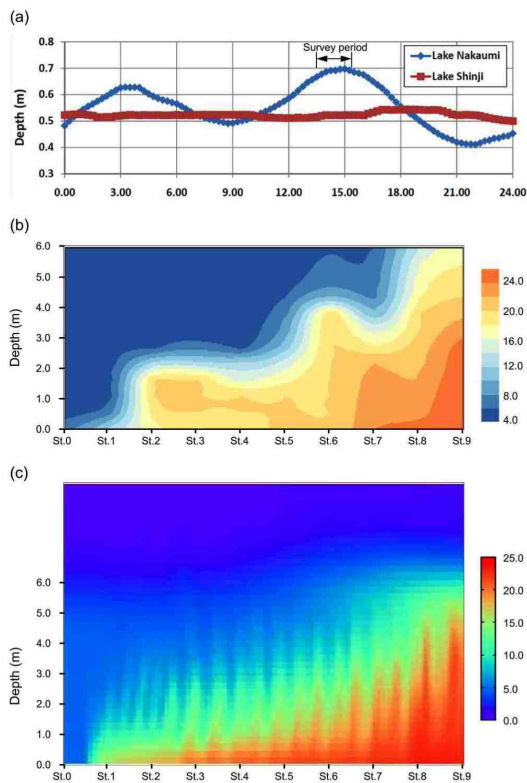


Fig. 9 Comparison of the measured and the estimated results at 14:30 Pm: (a) water level fluctuation, (b) measured salinity values along the Ohashi River, and (c) estimated salinity values along the Ohashi River.

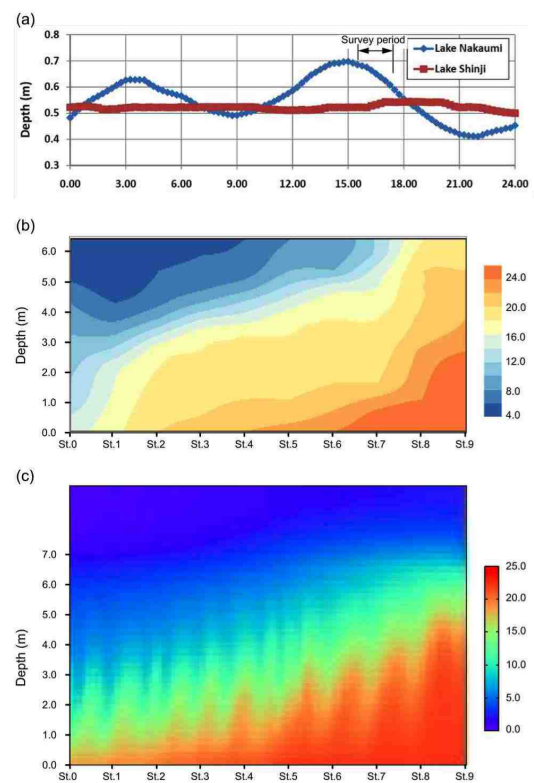


Fig. 10 Comparison of the measured and the estimated results at 16:30 Pm: (a) water level fluctuation, (b) measured salinity values along the Ohashi River, and (c) estimated salinity values along the Ohashi River.

5. Conclusion

A two-dimensional multiphase numerical model for incompressible, immiscible and variable density fluids has been developed to study salinity propagation of brackish coastal lakes due to tidal water flux. The governing equations are discretized in a collocated grid system and solved with a finite volume method. Using openMp parallel programming approach reasonably decreased the computational time. The developed model successfully predicted the density current propagation in the lock-exchange problems for Boussinesq and non-Boussinesq experimental cases. In addition, saltwater intrusion into Ohashi River was simulated and validated using field measured data. The predicted model still needs refinement to avoid concentration oscillations.

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