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Generalized Scaling Relations for Level Ground Response

Tetsuo TOBITA, Susumu IAI and Saki NODA

Synopsis

To investigate the generalized scaling relation in centrifuge modeling, a prototype is scaled down to 1/100 with 9 combinations of scaling factors of virtual 1 G and centrifugal field. The model ground is flat and made of a homogeneous sand layer. Five accelerometers are employed in various depths. Dynamic input motions are scaled accordingly. In prototype scale, the applicability of the scaling relation is evaluated by examining the identity of dynamic responses obtained from 9 cases. Results show that shear wave velocities are approximately the same value and, therefore, the generalized scaling relation of shear wave velocity is confirmed. For the scaling relation of acceleration, when the ground response is nearly elastic, the scaling law is confirmed for a range of centrifugal acceleration applied in this study.

Keywords: Centrifuge model testing, scaling relation, dynamic,

1. Introduction

The size of physical model is increasing with demands from earthquake engineering community for rigorous investigation on structure’s ultimate state. For example, the world largest shaking table of 20 × 15 m has been built in the E-defense, Japan. It can shake a real scale 6-story reinforced concrete building (1,000 t) (Chen et al. 2006), or 2 wooden Japanese houses simultaneously (Suzuki et al. 2006). However, even with such a large shaking table, when dynamic behavior of a whole structure including its foundation buried into the ground is examined, a prototype has to be scaled down due to limitations of shaking table’s capacity (Tokimatsu et al. 2007).

In centrifuge modeling, geometrical scale of a model can be theoretically decreased by increasing the centrifugal acceleration. However, with decreasing model scale, the problem of scaling effects, i.e., dependence of model behavior on a relative size of structure and granular material, becomes more and more apparent (e.g., Honda and Towhata 2006). Other problems for dynamic testing under larger centrifugal acceleration are the requirements of more powerful actuator and its precise control (Chazelas et al. 2006).

To overcome these deficiency in centrifuge tests and increase the efficiency of small to medium size geotechnical centrifuges, two stage scaling relationship called generalized scaling relationship for centrifuge tests was proposed by Iai et al. (2005) (Figure 1). In this scaling relation, recorded physical model parameters are converted to those in the virtual 1G field with scaling factor for centrifuge model tests, η [Fig. 1(a)], then the parameters are further converted to prototype with scaling factor for 1G tests, μ [Fig. 1(b)] (Iai 1989). By using this scaling relationship, model tests with scaling factor (prototype/physical model) of 100 or much higher may be possible.

Tobita and Iai (2007) studied the applicability of the scaling law with pile foundations. However, they encountered some difficulties concerned with precise control of shake table required for rigorous investigations. In the present study, a newly equipped shake table is employed. In the experiments, a prototype is scaled down to 1/100

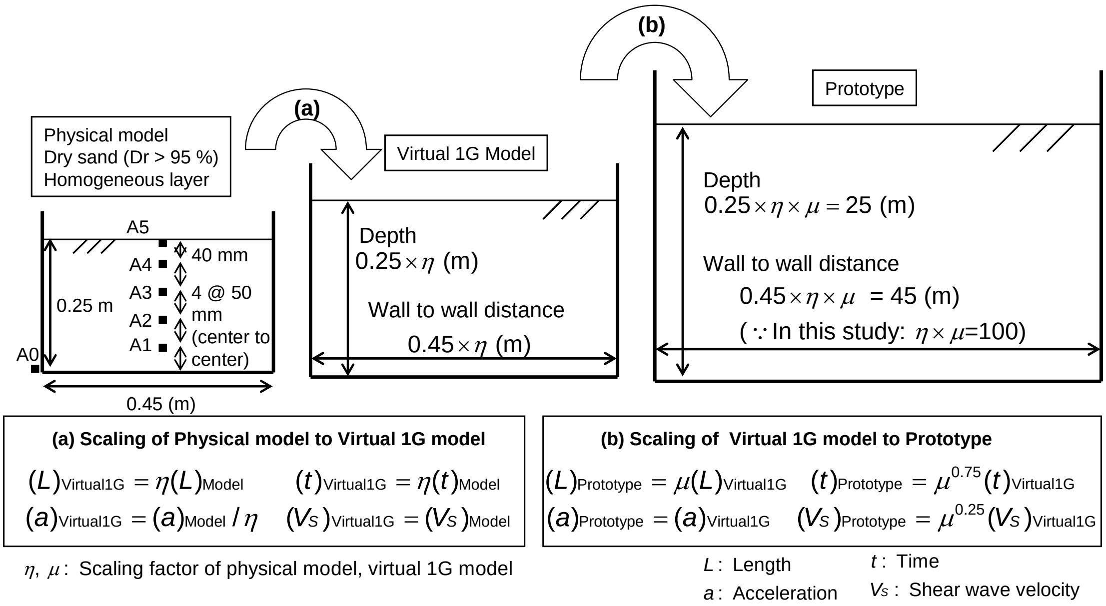

Figure 1. Physical model setup and concept of the two stage scaling with associated scaling relationship: (a) scaling relations for centrifugal field and (b) scaling relations for 1G field.

with 9 combinations of scaling factors of virtual 1 G and centrifugal field. Input motions are also scaled accordingly. Then the generalized scaling relation is examined by comparing dynamic responses in the prototype scale. If the generalized scaling law is valid, those responses are identical regardless of scaling factors. In the present paper, only 4 out of 9 cases, and cases of the smallest input motion are mainly discussed.

2. Generalized scaling relationship

This section briefly reviews the derivation of generalized scaling relationship (Iai et al. 2005) of physical model tests based on the fundamental physical laws, for example, stress equilibrium, definition of strains, and a constitutive relation.

Stress equilibrium:

$$\partial\sigma_{ij,j} + X_i = \rho\ddot{u}_i \qquad (1)$$

Definition of strain:

$$\varepsilon_{ij} = \left(u_{i,j} + u_{j,i}\right)/2 \qquad (2)$$

Constitutive relation:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \qquad (3)$$

where σ_{ij} is stress tensor, x_i is coordinate system, ρ is density, $\ddot{u}_i$ is acceleration and dots mean temporal differentiation and $X_i = (0,\ -\rho g,\ 0)$, g is acceleration due to gravity, ε_{ij} is strain tensor and C_{ijkl} is tangential stiffness modulus. Here, the summation rule is supposed.

The scaling relations for centrifuge model tests are derived by introducing scaling factors for variables appearing in equations (1) - (3) as follows and by demanding that these variables must satisfy both the equations for prototype and the model.

$$(x_i)_p = \lambda(x_i)_m,\ (\sigma_{ij})_p = \lambda_\sigma(\sigma_{ij})_m,\ (u_i)_p = \lambda_u(u_i)_m,$$

$$(\rho)_p = \lambda_\rho(\rho)_m,\ (g)_p = \lambda_g(g)_m,\ (\varepsilon_{ij})_p = \lambda_\varepsilon(\varepsilon_{ij})_m,$$

$$(t)_p = \lambda_t(t)_m,\ (C_{ijkl})_p = \lambda_C(\varepsilon_{ijkl})_m$$

where subscripts "*p*" and "*m*" mean, respectively, "prototype" and "model." By substituting variables for prototype into Eq. (1),

$$(\sigma_{ij.j})_p + (X_i)_p = (\rho)_p(\ddot{u}_{ij})_p \qquad (4)$$

Then introducing scaling relations into Eq. (4),

$$\lambda_\sigma/\lambda(\sigma_{ij,j})_m + \lambda_\rho\lambda_g(X_i)_m = \lambda_\rho\lambda_u/\lambda_t^2(\rho)_m(\ddot{u}_{ij})_m \qquad (5)$$

Since variables for model also satisfy Eq. (1), then all the coefficients of Eq. (5) must be equal as follows,

$$\lambda_\sigma/\lambda = \lambda_\rho\lambda_g = \lambda_\rho\lambda_u/\lambda_t^2 \qquad (6)$$

Now, from the left hand side of Eq. (6), the scaling relation of stress is written as,

$$\lambda_\sigma = \lambda\lambda_\rho\lambda_g \qquad (7)$$

From Eq. (2), (3) and (6) in the same way, the scaling relation of time, displacement and stiffness

Table 1. Generalized scaling factors for centrifuge model tests ($\mu_\varepsilon = \mu^{0.5}$) (Iai et al. 2005)

	Partitioned		Generalised
	Centrifugal field	Virtual 1G field	
	η=Prototype /physical model	μ=Prototype /virtual model	Prototype /physical moc
Length	η	μ	$\mu\eta$
Density	1	1	1
Time	η	$\mu^{0.75}$	$\mu^{0.75}\eta$
Stress	1	μ	μ
Pore water pressure	1	μ	μ
Displacement	η	$\mu^{1.5}$	$\mu^{1.5}\eta$
Particle velocity	1	$\mu^{0.75}$	$\mu^{0.75}$
Shear wave velocity	1	$\mu^{0.25}$	$\mu^{0.25}$
Acceleration	$1/\eta$	1	$1/\eta$
Strain	1	$\mu^{0.5}$	$\mu^{0.5}$
Bending moment	$\eta^{3.0}$	$\mu^{4.0}$	$\mu^{4.0}\eta^{3.0}$
Flexial rigidity	$\eta^{4.0}$	$\mu^{4.5}$	$\mu^{4.5}\eta^{4.0}$

are given by,

$$\lambda_t = \left(\lambda\lambda_\varepsilon / \lambda_g\right)^{0.5} \quad , \quad \lambda_u = \lambda\lambda_\varepsilon \quad , \quad \lambda_C = \lambda\lambda_\rho\lambda_g / \lambda_\varepsilon \tag{8}$$

Now let us partition the scaling factors for length, density, acceleration, and strain as follows,

$$\lambda = \eta\mu \,,\ \ \lambda_\rho = \eta_\rho\mu_\rho \,,\ \ \lambda_g = \eta_g\mu_g \,,\ \ \lambda_\varepsilon = \eta_\varepsilon\mu_\varepsilon \tag{9}$$

where η and μ denote respectively the scaling factor of length for centrifuge and 1 g model tests. The value of the scaling factor for acceleration due to gravity in 1 g field is unity ($\mu_\sigma=1$) and that for centrifugal field is $\eta_g = 1/\eta$. The scaling factor for density and strain in centrifugal field are $\eta_\sigma = \mu_\varepsilon = 1$. Substituting these into the above relations yields the generalized scaling relationship,

$$\lambda = \eta\mu \,,\ \ \lambda_\rho = \mu_\rho \,,\ \ \lambda_g = 1/\eta \,,\ \ \lambda_\varepsilon = \mu_\varepsilon \tag{10}$$

In general, scaling relation of shear wave velocity can be derived as follows by using the shear wave velocity of the model ground, $(V_s)_m$, and that of the prototype ground, $(V_s)_p$. Shear modulus at small strain, of the model ground $(G_0)_m$ and the prototype ground $(G_0)_p$ are expressed,

$$(G_0)_m = (\rho)_m (V_S)_m^2 \tag{11}$$

$$(G_0)_p = (\rho)_p (V_S)_p^2 \tag{12}$$

These moduli give the scaling factor for the tangent modulus of soil as,

$$\begin{aligned}\lambda_C &= [(\rho)_p (V_S)_p^2]/[(\rho)_m (V_S)_m^2] \\ &= \lambda_\rho [(V_S)_p / (V_S)_m]^2\end{aligned} \tag{13}$$

whereas the similitude of shear modulus is $\lambda_C = \lambda\lambda_\rho\lambda_g / \lambda_\varepsilon$ (Eq. 8). Consequently, the scaling factor for the strain is given by,

$$\lambda_\varepsilon = \lambda\lambda_g / [(V_s)_p / (V_s)_m]^2 \tag{14}$$

Table 2. Scaling factors applied in the present study

	Scaling factor		
	Centrifugal field	Virtual 1G field	Prototype
Case	η	μ	$\mu\eta$
1G	1	100	
8G	8	12.5	
10G	10	10	
20G	20	5	
30G	30	3.33	100
40G	40	2.5	
50G	50	2	
60G	60	1.67	
70G	70	1.43	

Therefore, the scaling relation of shear wave velocity is given by,

$$\begin{aligned}\lambda_{Vs} &= (V_s)_p / (V_s)_m = \sqrt{\lambda\lambda_g / \lambda_\varepsilon} \\ &= \sqrt{(\eta\mu)(\eta_g\mu_g)/\mu^{1-N}} = \sqrt{(\eta\mu)(1/\eta)/\mu^{1-N}} = \mu^{N/2}\end{aligned} \tag{15}$$

where the scaling factor of strain is assumed to be $\mu_\varepsilon=\mu^{1\text{-}N}$. The generalized scaling relationships are summarized in Table 1 with the scaling factor of density and strain $\mu_\sigma=1$ and $\mu_\sigma=\mu^{0.5}$ (i.e., N=0.5) in 1 g field (Iai 1989). Note that the scaling factor of particle velocity, $\mu^{0.75}$ is different from that of shear wave velocity, $\mu^{0.25}$ in 1g field.

3. Centrifuge model tests and investigation of the generalized scaling law

The experiments were conducted in a rigid wall container mounted on 2.5 m radius geotechnical centrifuge at the Disaster Prevention Research Institute, Kyoto University (DPRI-KU). Overall dimensions of the rigid container are 450 × 150 × 300 mm in length, width, and height, respectively. Dynamic excitation was given in the direction parallel to the cross-section shown in Figure 1 by a shake table mounted on a platform. The shake table was controlled by displacement signals. An accelerometer was attached to the base plate of the shake table to measure input motion. Five accelerometers were installed in the model ground of compacted dry silica sand (e_{max}=1.19, e_{min}=0.71, and D_{50}=0.15 mm) with relative density more than 95% (Figure 1). To obtain firm model ground, dry tamping method was employed.

As shown in Table 2, total 9 cases with various scaling factors of length, η and μ were considered. Since the model ground was well compacted, the experiments were consecutively carried out from small to large centrifugal acceleration. The scaling factors of centrifugal field, η, correspond to the centrifugal acceleration, while the scaling factors of the virtual 1 G field, μ are selected so that the

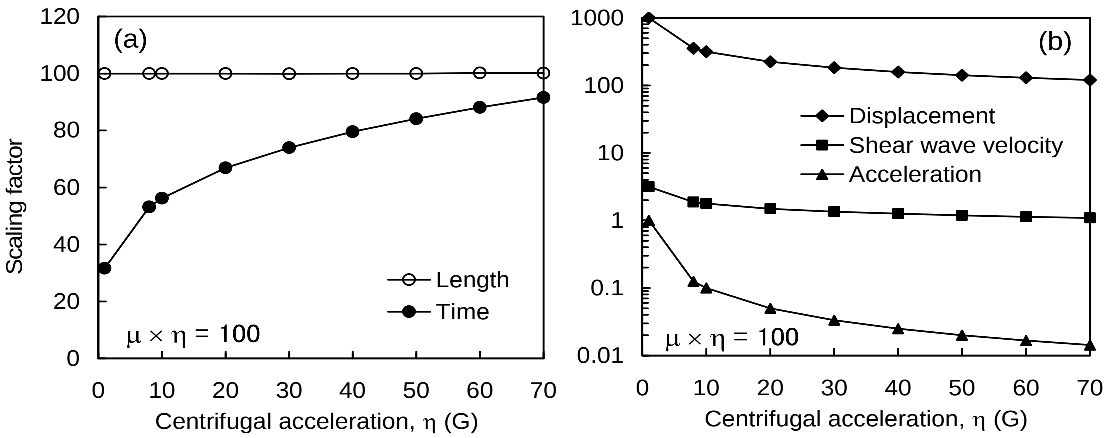

Figure 2. Scaling factors of length and time (a), displacement, shear wave velocity and acceleration (b) for model tests conducted in the present study.

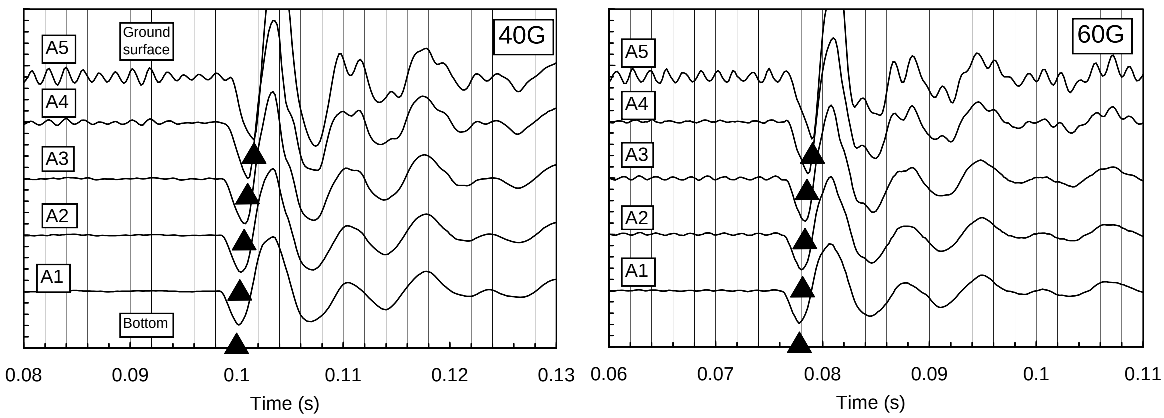

Figure 3. Time histories of response acceleration against impulsive input motion and arrival time of the 1st peak specified with solid triangle for Cases 40 G and 60 G (in model scale).

scaling factor of prototype, $\eta \times \mu$, is equal to 100. Other scaling factors, time, shear wave velocity, displacement and acceleration for each centrifugal acceleration are given in Figure 2 together with the scaling factor of length whose value is constant, i.e., $\eta \times \mu = 100$. As shown in Fig. 2(b), the scaling factor of shear wave velocity is rather insensitive to centrifugal acceleration (it varies from 1 to 3 for a range of 1 G to 70 G), while that of acceleration and displacement are sensitive to centrifugal acceleration. Scaling factor of acceleration varies from 1 to 0.014 in a range of centrifugal acceleration of 1 G to 70 G, and that of displacement from 1000 to 120 in the same range of centrifugal acceleration. The scaling factor of time varies from 31 to 91 in a range of 1 G to 70 G.

To evaluate scaling relationship of the shear wave velocity, travel time of impulsive input motion (single sin wave with 250 Hz in model scale) was measured. The travel time in this study was taken as the arrival of the 1st peak due to a difficulty encountered to specify exact arrival time of signals. Based on the time histories of acceleration, such as shown in Fig. 3 for cases 40 G and 60 G, shear wave velocities in the model scale were derived [Fig. 4(a)], then, by using scaling factors shown in Fig. 2(b), they were converted to the prototype scale [Fig. 4(b)]. Shear wave velocities with different markers shown in Fig. 4 are derived by the difference of distance and travel time between sensors A3 to A5 and A1. Travel time of A2 was not used because time difference between A1 and A2 was too small to be captured by the sampling frequency employed in the tests (5 kHz). In model scale, shear wave velocities tend to increase as centrifugal acceleration increase [Fig.

Table 3. Input frequencies for sinusoidal waves

Case	Frequency (Hz) Centrifugal field	Prototype
1G	20.6	
8G	34.6	
10G	36.6	
20G	43.5	
30G	48.1	0.65
40G	51.7	
50G	54.7	
60G	57.3	
70G	59.5	

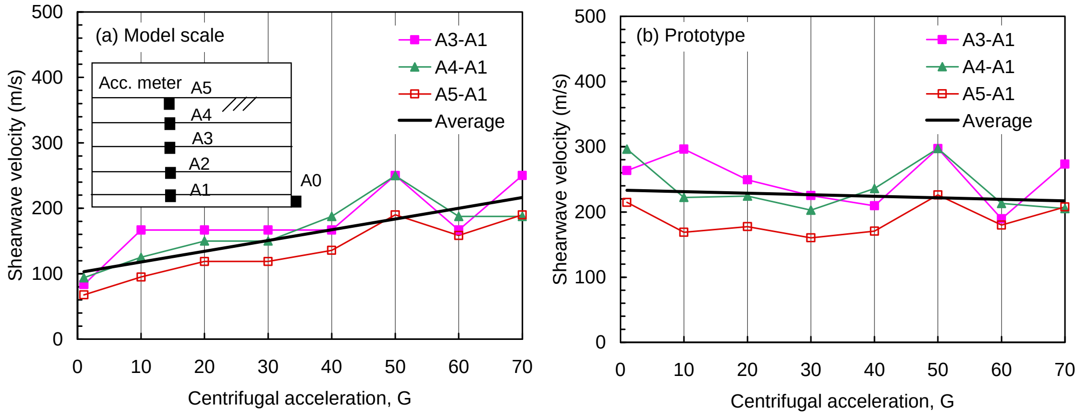

Figure 4. Shear wave velocities in model scale (a), and prototype (b)

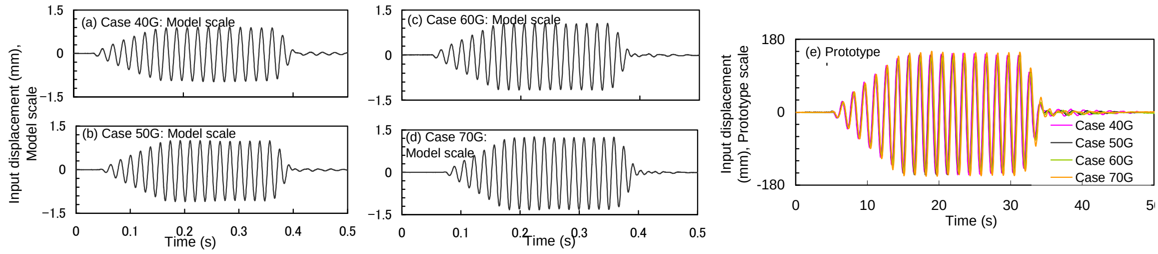

Figure 5. Time histories of input displacements of Case 40G (a), 50G (b), 60G (c), and 70G (d) in model scale, and all cases combined in prototype scale (e).

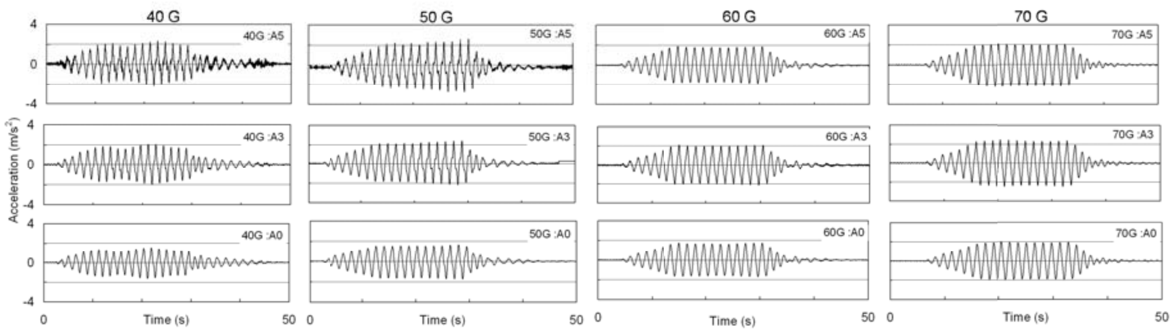

Figure 6. Time histories of input (A0) and response (A3 and A5) acceleration of Cases 40 G to 70G.

4(a)], while, in prototype scale [Fig. 4(b)], shear wave velocity becomes more or less constant, about 230 m/s on average.

Next, to investigate the scaling law of acceleration, the model was excited by sinusoidal input motions (0.65 Hz, duration 35 s in prototype scale) (Table 3). Figures 5(a) to (d) are the time histories of input displacements in model scale and Fig. 5(e) is the converted time history in prototype scale. A range of displacement amplitude is from 0.9 mm to 1.2 mm in model scale. After conversion, the amplitude becomes 150 mm in prototype scale. As shown in Fig. 5(e), similar input motions were employed in all cases. Time histories of acceleration recorded at the base (A0), in the middle layer (A3), and at the ground surface (A5) for Cases 40 G to 70 G are plotted in Fig. 6. These are all in prototype scale. As seen in Fig. 6, all the input and response acceleration amplitude except for Case 40 G are about 2 m/s^2 indicating the response may be in a linear elastic range. In this range, the generalized scaling law of acceleration under the centrifugal acceleration of 50 G up to 70 G is validated. For Case 40 G, input acceleration amplitude is reduced to about 1 m/s^2. This might be due to the mechanical resonance of the centrifuge equipment used in the present study as seen in Fig. 6 (Case 40 G) with lasting vibration after the end of shaking. The other possibility is the sensitivity of scaling factor of acceleration to centrifugal acceleration shown in Fig. 2(b). Considering other tests cases with lower centrifugal acceleration, the applicability of the generalized scaling relation is largely confirmed.

4. Conclusions

Applicability of the generalized scaling law for centrifuge modeling is investigated. In the present study, a prototype is scaled down to 1/100 with 9 combinations of scaling factors of virtual 1 G and centrifugal field. Input motions are also scaled accordingly. Four out of 9 cases with the smallest input motions are mainly discussed. The generalized scaling relation is investigated by comparing responses in the prototype scale. Prototype shear wave velocities were close each other and the generalized scaling law of shear wave velocity was confirmed. For the scaling law of acceleration, when the ground response was nearly linear elastic, the scaling law was confirmed with centrifugal acceleration of 50 G up to 70G. Considering other tests cases with lower centrifugal acceleration, the applicability of the generalized scaling relation is largely confirmed.

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水平成層地盤における拡張型相似則の検証

飛田哲男・井合進・野田早紀

要　旨

近年，実験模型の大型化が進んでいる。しかし，地盤−基礎構造物の相互作用問題について実大模型を作成し実験を行うことは，発破による液状化試験など特殊な事例を除き，現段階では不可能に近い。このため，遠心模型実験が用いられることが多いが，中小型の遠心模型実験装置では装置の容量や使用できる土槽の大きさなどによる制約がある。そこで，Iaiら(2005)は仮想的な1G場模型を考え，それをターゲットとして遠心模型実験を行い，実験結果に対し遠心場の模型相似則と1G場の模型相似則(Iai 1989)を連続して適用し実物スケールに換算する相似則を提案した。ここでは，これを「拡張型相似則」と呼ぶ。

キーワード:遠心模型実験，相似則，動的載荷