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Verification of Generalized Scaling Relations for Dynamic Centrifuge Experiments

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Synopsis

Dynamic centrifuge tests for pile foundations are conducted to investigate the applicability of the generalized scaling relation proposed by Iai et al. (2005). In the centrifuge tests, geometrical scale of a model pile foundation (prototype/centrifuge model) is set as 100 (= $\mu\eta$). Five combinations of scaling factors of virtual 1 g model (μ) and centrifuge model (η) are tested. According to the combination of the scaling factors, values of flexural rigidity of a pile are determined in centrifuge model. Responses in prototype scale are compared each other for five pile foundation models tested under various centrifugal accelerations. Fairly good agreements are obtained for the amplitude of input displacement and input acceleration for all the cases, except the case of large input motion. Also for the case of lower centrifugal accelerations, the agreements are significant for the average amplitude of acceleration in soil. Not only the average amplitude of responses, but also bending moment profile of lower centrifugal accelerations shows faire agreements to justify the applicability of the generalized scaling relation. When centrifugal acceleration is larger, some responses become small and the exact cause of this is unknown yet.

Keywords: similitude law, centrifuge model tests, soil structure interaction

1. Introduction

With recent demand from earthquake engineering community to carry out physical model testing of larger prototypes, a size of experimental facility is becoming larger and larger. For example, the world largest shaking table of 20 × 15 m has been built in the E-defense, Japan. It can shake a real scale 6-story reinforced concrete building (1,000 t) (Chen et al. 2006), or 2 wooden Japanese houses simultaneously (Suzuki et al. 2006). However, even with such a large shaking table, when dynamic behavior of a whole structure including its foundation buried into the ground is examined, a prototype has to be scaled down due to the limitation of the shaking table's capacity (Suzuki and Tokimatsu 2007).

In centrifuge modeling, geometrical scale of a model can be theoretically decreased by increasing the centrifugal acceleration. However, with decreasing model scale, the problem of scaling effects, i.e., dependence of model behavior on a relative size of structure and granular material (e.g., Honda and Towhata 2006), becomes more and more apparent. Other problem for dynamic testing under larger centrifugal acceleration is the requirement of more powerful actuator and its precise control (Chazelas et al. 2006).

To overcome these deficiency in centrifuge tests and increase the efficiency of small to medium size centrifuges, two stage scaling relationship called generalized scaling relationship for centrifuge tests was proposed by Iai et al. (2005) (Fig. 1). In this scaling relation, a prototype is scaled down to a 1 g model with scaling factor for 1 g model tests (Iai 1989), and the 1 g model is further scaled down to a centrifuge model with scaling factor for centrifuge model tests. By using this scaling relationship, model tests with scaling factor (prototype/physical model) of 100 or much higher may be possible.

In the present study, using a prototype and five centrifuge models scaled down to 1/100 with various scaling factors for 1 g and centrifuge model tests, the generalized scaling relation is investigated by comparing the responses of the five models in prototype scale.

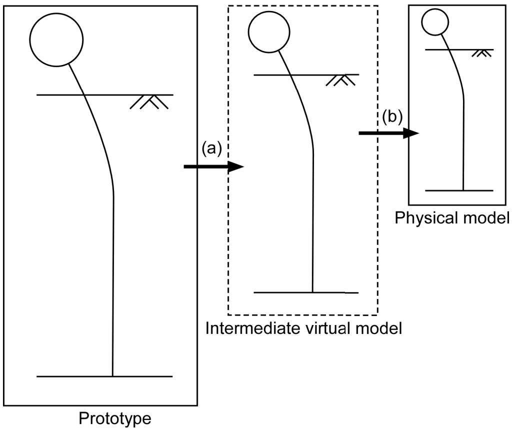

Fig. 1 Concept of two stage scaling: (a) scaling relations for 1g field; (b) scaling relations for centrifugal field (modified after Iai et al., 2005).

2. Generalized scaling relationship

This section briefly reviews the derivation of generalized scaling relationship (Iai et al. 2005) of physical model tests based on the fundamental physical laws, for example, stress equilibrium, definition of strains, and a constitutive relation.

Stress equilibrium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + X_i = \rho \ddot{u}_i \tag{1}$$

Definition of strain:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

Constitutive relation:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{3}$$

where σ_{ij} is stress tensor, x_i is coordinate system, ρ is density, $\ddot{u}_i$ is acceleration and dots mean temporal differentiation and $X_i = (0,\ -\rho g,\ 0)$, g is acceleration due to gravity, ε_{ij} is strain tensor and C_{ijkl} is tangential stiffness tensor. Here, the summation rule is supposed.

The scaling relations for centrifuge model tests are derived by introducing scaling factors for variables appearing in equations (1) - (3) as follows and by demanding that these variables must satisfy both the equations for prototype and the model.

$$(x_i)_p = \lambda (x_i)_m,\ (\sigma_{ij})_p = \lambda_\sigma (\sigma_{ij})_m,\ (u_i)_p = \lambda_u (u_i)_m,$$

$$(\rho)_p = \lambda_\rho (\rho)_m,\ (g)_p = \lambda_g (g)_m,\ (\varepsilon_{ij})_p = \lambda_\varepsilon (\varepsilon_{ij})_m,$$

$$(t)_p = \lambda_t (t)_m,\ (C_{ijkl})_p = \lambda_C (\varepsilon_{ijkl})_m$$

where subscripts "*p*" and "*m*" mean, respectively, "prototype" and "model." By substituting variables for prototype into Eq. (1),

$$\frac{\partial (\sigma_{ij})_p}{\partial (x_j)_p} + (X_i)_p = (\rho)_p \frac{\partial^2 (u_{ij})_p}{\partial (t)_p^2} \tag{4}$$

Then introducing scaling relations into Eq. (4),

$$\frac{\lambda_\sigma}{\lambda}\frac{\partial (\sigma_{ij})_m}{\partial (x_j)_m} + \lambda_\rho \lambda_g (X_i)_m = \lambda_\rho \frac{\lambda_u}{\lambda_t^2}(\rho)_m \frac{\partial^2 (u_{ij})_m}{\partial (t)_m^2} \tag{5}$$

Since variables for model also satisfy Eq. (1), then all the coefficients of Eq. (5) must be equal as follows,

$$\lambda_\sigma / \lambda = \lambda_\rho \lambda_g = \lambda_\rho \lambda_u / \lambda_t^2 \tag{6}$$

Now, from the left hand side of Eq. (6), the scaling relation of stress is written as,

$$\lambda_\sigma = \lambda \lambda_\rho \lambda_g \tag{7}$$

From Eq. (2), (3) and (6) in the same way, the scaling relation of time, displacement and stiffness are given as,

$$\lambda_t = \left(\lambda \lambda_\varepsilon / \lambda_g\right)^{0.5},\ \lambda_u = \lambda \lambda_\varepsilon,\ \lambda_C = \lambda \lambda_\rho \lambda_g / \lambda_\varepsilon .$$

Now let us partition the scaling factors for length, density, acceleration, and strain as follows,

$$\lambda = \mu \eta,\ \lambda_\rho = \mu_\rho \eta_\rho,\ \lambda_g = \mu_g \eta_g,\ \lambda_\varepsilon = \mu_\varepsilon \eta_\varepsilon$$

where η and μ denote respectively the scaling factor of length for 1 g and centrifuge model tests. The value of the scaling factor for acceleration due to gravity in 1 g field is unity ($\mu_g = 1$) and that for centrifugal field is $\eta_g = 1/\eta$. The scaling factor for density and strain in centrifugal field are $\eta_\rho = \eta_\varepsilon = 1$. Substituting these into the above relations yields the generalized scaling relationship,

$$\lambda = \mu\eta,\ \lambda_\rho = \mu_\rho,\ \lambda_g = 1/\eta,\ \lambda_\varepsilon = \mu_\varepsilon$$

The generalized scaling relationships used in the present study are summarized in Table 1 with the scaling factor of density and strain $\mu_\rho = 1$ and $\mu_\varepsilon = \mu^{0.5}$ in 1 g field (Iai 1989).

Table 1 Generalized scaling factors for centrifuge model tests (Iai et al. 2005)

	Partitioned		Generalised
	Virtual 1G field μ=Prototype /virtual model	Centrifugal field η=Prototype /physical model	Prototype /physical model
Length	μ	η	$\mu\eta$
Density	1	1	1
Time	$\mu^{0.75}$	η	$\mu^{0.75}\eta$
Stress	μ	1	μ
Pore water pressure	μ	1	μ
Displacement	$\mu^{1.5}$	η	$\mu^{1.5}\eta$
Velocity	$\mu^{0.75}$	1	$\mu^{0.75}$
Acceleration	1	$1/\eta$	$1/\eta$
Strain	$\mu^{0.5}$	1	$\mu^{0.5}$
Bending moment	$\mu^{4.0}$	$\eta^{3.0}$	$\mu^{4.0}\eta^{3.0}$
Flexial rigidity	$\mu^{4.5}$	$\eta^{4.0}$	$\mu^{4.5}\eta^{4.0}$

3. Configuration of centrifuge tests

Experiments were conducted in a rigid wall container mounted on 2.5 m radius geotechnical centrifuge at the Disaster Prevention Research Institute, Kyoto University (DPRI-KU). Overall dimensions of the rigid container are 450 × 150 × 300 mm in length, width, and height, respectively. Dynamic excitation was given in the direction parallel to the cross-section shown in Fig. 2. A shake table mounted on a platform was unidirectionally driven by a servo hydraulic actuator.

A vertical cross-section of the pile foundation model in a deposit of dry sand is depicted in Fig. 2. Sand deposit (silica sand: e_{max} = 1.01, e_{min} = 0.76, and D_{50} = 0.5 mm) was prepared by the air pluviation to a target relative density of 70 % in 250 mm lifts (model scale). The length of a pile was 300 mm (model scale) and a mass of 0.5 kg (model scale) was put on top of the foundation. Rotation was fixed on both top and bottom of piles. Ten pairs of strain gauges were attached at the specified height of the pile (Fig. 2). As shown in Fig. 2, the model was instrumented with four accelerometers, one laser displacement transducers. Sampling frequency was 5 kHz.

To investigate the generalized scaling relations, five individual pile foundation models were used. Models were designed so that the flexural rigidities, *EI*, were identical each other in prototype scale. The horizontal cross section of a model pile is rectangular and their thickness in the shaking direction is varied with the scaling factors.

Scaling factor of prototype/centrifuge model was set as 100 (= $\mu\eta$), and the scaling factors of 1 g (μ) and centrifugal (η) fields were varied with 5 patterns as shown in Table 2. Input frequency of prototype was set as 1.0 Hz, and by applying the generalized scaling factors, frequencies of input displacement in centrifugal fields were determined as shown in Table 3. In each case, three consecutive input displacements (amplitudes of 51, 72 and 97mm in prototype scale) were applied at the base of models. Converted input displacement amplitudes to the centrifugal fields are shown in Table 4. The input displacement (Fig. 3) is tapered off in the beginning and at the end and has 20 s (prototype) of flat amplitude in between.

Table 2 Scaling factors of virtual 1 g field and centrifugal field used in the experiments.

Case	Virtual 1G field μ	Centrifugal field η
1	2.1	48.1
2	2.6	38.5
3	5.2	19.2
4	10.4	9.6
5	20.8	4.8

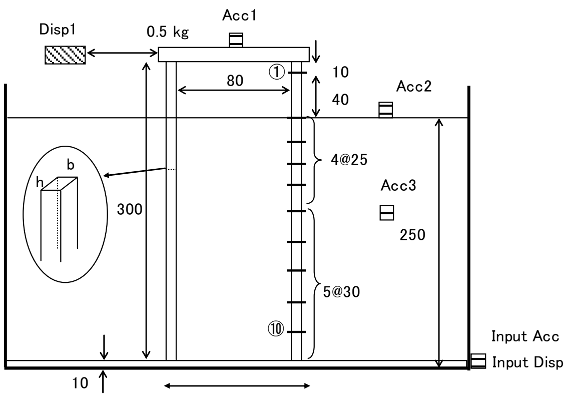

Fig. 2 Cross section of centrifuge model (Units are in mm: model scale)

Table 3 Input frequency of prototype, virtual 1 g field and centrifugal field.

Case	Input frequency (Hz) Prototype	Virtual 1G field	Centrifugal field
1		1.7	83.3
2		2.0	78.8
3	1.0	3.4	66.2
4		5.8	55.7
5		9.8	46.8

Table 4 Average input displacement of prototype, virtual 1 g field and centrifuge field.

Case	Amplitude of input displacement (mm) Prototype	Virtual 1G field	Centrifugal field
1		16.8	0.4
2		12.0	0.3
3	51.0	4.2	0.2
4		1.5	0.2
5		0.5	0.1
1		24.1	0.5
2		17.2	0.4
3	72.0	6.1	0.3
4		2.1	0.2
5		0.8	0.2
1		32.2	0.7
2		23.1	0.6
3	91.0	8.1	0.4
4		2.9	0.3
5		1.0	0.2

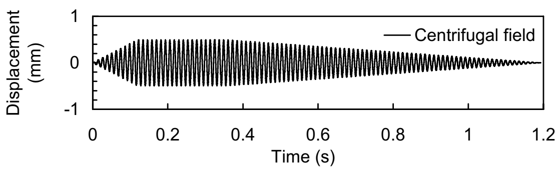

Fig. 3 Example of input displacement in centrifugal field.

4. Verification of generalized scaling relation

Data obtained by the centrifuge tests are converted into the ones in prototype scale by applying the generalized scaling relation. For all the test cases, corresponding responses are supposed to be identical as long as the input motions are the same. To confirm this, Fig. 4 compares average amplitudes of input displacements between specified and measured. "Specified" amplitudes are the target amplitude of each test. As shown in Table 4, these values are given as 51, 72, and 97 mm in prototype scale. "Measured" amplitudes are obtained by the gap sensor attached to the shaking table. In all cases in Fig. 4, measured amplitudes are more or less off from the specified amplitude. This is due to the lack of precise control of shaking table. However, fairly good agreements are obtained for all the cases except 97 mm input of Case 2 that is 1.5 times larger than the specified amplitude.

The generalized scaling relations can be evaluated by comparing the average amplitude of measured responses. Prototype responses are compared in Fig. 5(a-d) whose horizontal axis is the case number and vertical axis is the average value of measured response. If the scaling is correct, measured responses of each specified input displacement in all the cases coincide. In Fig. 5(a), for the input displacement of 51 and 72 mm, almost identical amplitude is obtained for the measured input displacement. For the input acceleration shown in Fig. 5(b), average value is nearly the same except Case 2 that is slightly higher than the others. Average acceleration of Acc3 [Fig. 5(c)] for the input displacements of 51 and 72 mm of Case 3 to 5 shows fairly good agreements. However, for Case 1 and 2, the response is significantly lower than the others. One possible cause of this degradation is that the ground is vibrating with higher mode because input frequency in centrifugal field is 83.3 and 78.8 Hz for Case 1 and 2, respectively (Table 3). However, its effect was not clearly seen in time and frequency domain, and, therefore, thorough investigation is required. Average pile head displacements are more scattered than the other responses shown in Fig. 5(a)-(c) except Case 3 to 5 of 72 mm of input displacement [Fig. 5(d): (triangle makers)] which show almost the same level of response.

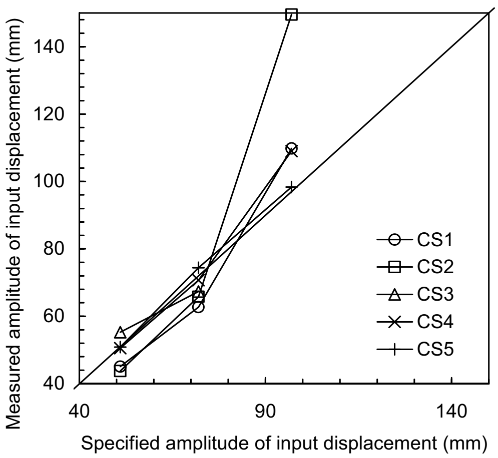

Fig. 4 Specified versus measured amplitude of input displacement. Amplitude is an average.

Fig. 6 shows vertical profiles of bending moment when the pile head displacement is absolute maximum. Profiles of Case 3 to 5 are similar in that the absolute maximum of bending moment is obtained at the depth

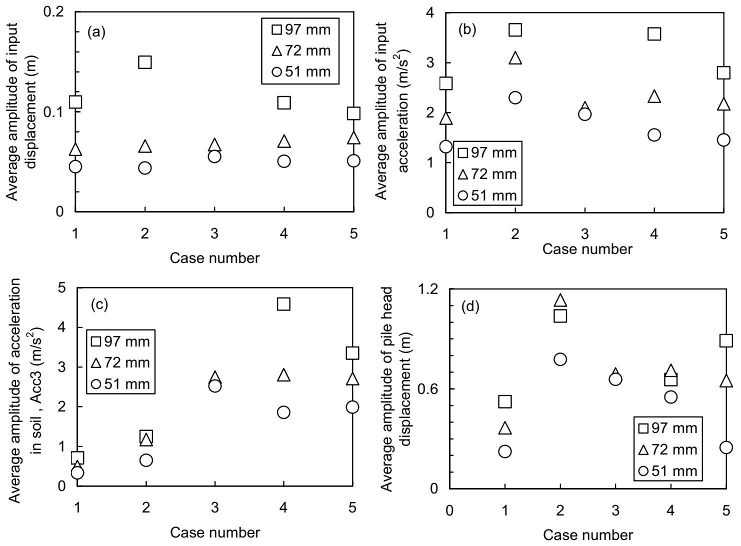

Fig. 5 Average amplitude of input displacement (a), input acceleration (b), acceleration in soil, Acc3 (c), and pile head displacement (d).

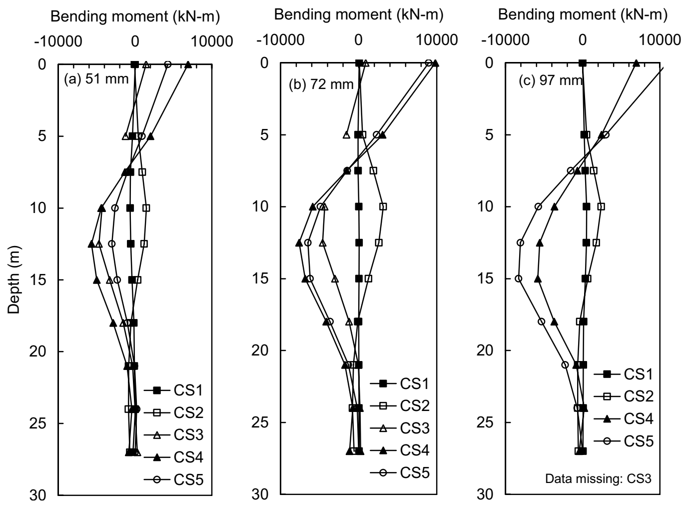

Fig. 6 Vertical profile of bending moment when the pile head displacement is absolute maximum.

of 13 m, while the profile of Case 1 is almost flat and of Case 2 shows reverse trend. When the input displacement becomes larger as shown in Fig. 6(b) and (c), the profile of Case 4 and 5 approaches each other verifying the applicability of the generalized scaling relations.

To investigate the low amplitude of bending moment of Case 1, the horizontal scale of the profile of 72 mm of input displacement is enlarged [Fig. 7(a)]. Also in Fig. 7, plotted are the time histories of pile head displacement [Fig. 7(b)] and bending moment at the depth of 10 m (Moment 4) [Fig. 7(c)]. In these time histories, square markers indicate the time when the bending moment profile is plotted. In Fig. 7, the profile with solid-square corresponds to the one at 20 s, while the one with solid-triangle is obtained by averaging over the period of 20 to 30 s. This profile shows slightly different shape of moment distribution in depth compared with Case 4 or 5 shown in Fig. 6. The shape of moment distribution of Case 1 [Fig. 7(a)] implies the incidence of higher mode of vibration in the model pile foundation. However, there was no evidence of the higher mode on records in both time and frequency domain.

5. Conclusions

Comparing the results obtained through centrifuge tests with various combination of scaling factors, the generalized scaling relation for dynamic centrifuge tests proposed by (Iai et al. 2005) were verified conditionally. In the centrifuge tests, geometrical scale

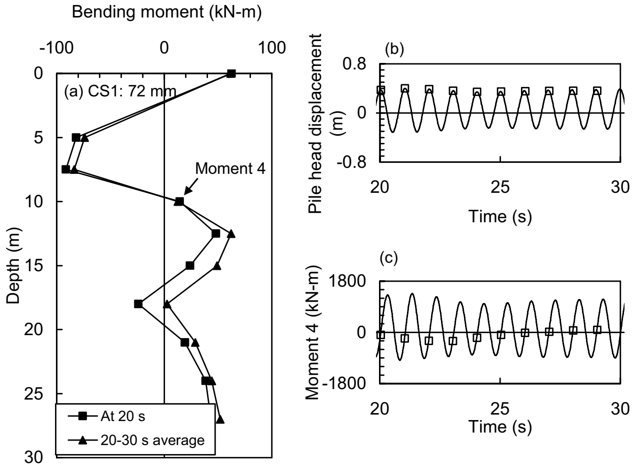

Fig. 7 Vertical profile of bending moment of CS1 (specified input displacement amplitude of 72 mm) (a) , time histories of pile head displacement (b) and bending moment of Moment 4 (c).

of a model pile foundation (prototype/centrifuge model) was 100 (= $\mu\,\eta$). Five combination of scaling factors of virtual 1 g model (μ) and centrifuge model (η) were tested. With the combination of the scaling factors, values of flexural rigidity of a pile were varied in centrifuge model. Responses in prototype scale were compared each other for five pile foundation models tested under various centrifugal accelerations.

Fairly good agreements are obtained for the amplitude of input displacement and input acceleration for all the cases, except the case of large input displacement (97 mm). Also for the case of lower centrifugal accelerations, agreements were significant for the average amplitude of acceleration in soil. Not only the average amplitude of response, but also bending moment profile of lower centrifugal accelerations shows faire agreements to justify the generalized scaling relation.

However, when the centrifugal acceleration is larger, the model ground as well as model piles might vibrate with higher mode. However, its effect was not clearly seen in time and frequency domain, and, therefore, thorough investigations on both modeling and centrifuge facility are required.

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動的遠心模型実験における拡張型相似則の検証

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要　旨

本研究では、井合ら(2005)が提案した拡張型相似則の検証を行う。地盤と構造物の動的相互作用問題について相似則の適用性を検討するため杭模型を用い、その曲げ剛性を相似則に従って5パターン変化させた実験を行う。計測データを実物スケールに変換したときに、対応する実験ケース間で振幅などの値が一致することをもって検証を行ったところ、遠心加速度が小さい場合には適用性が確認できた。

キーワード:遠心模型実験，拡張型相似則，動的相互作用