京都大学防災研究所年報 第 50 号 B 平成 19 年 4 月
Annuals of Disas. Prev. Res. Inst., Kyoto Univ., No. 50 B, 2007

The Spatio-Temporal Predictions of Rainfall-Sediment-Runoff Based on Lumping of a Physically-based Distributed Model

A P I P*, Takahiro SAYAMA, Yasuto TACHIKAWA* and Kaoru TAKARA

*Graduate School of Urban and Environment Engineering, Kyoto University

Synopsis

This paper proposes a lumping method of a distributed rainfall-sediment-runoff model. Based on the assumption of steady state conditions, the relationship between outflow discharges and water storage in a catchment can be derived. Then a lumped sediment model is developed. The maximum sediment storage in a catchment was mathematically derived as functions of sediment transport capacity and total storage of water at each grid-cell. Soil detachment and redeposition represented by the balance between the actual sediment storage and the maximum sediment storage in a catchment scale. The performance of lumped model is examined in the Lesti River, Indonesia.

Keywords: lumping, distributed rainfall-sediment-runoff model, erosion, Lesti River basin

1. Introduction

Estimations of the changes in total runoff and sediment yield with time in the catchment scale are required for solution of a number of problems. Design of dams and reservoirs, design of soil conservation, land-use planning, and water quality management are some of the examples. The annual sediment loads was correlated highly with sediment transported during the highest floods. Most of the material sediment is transported during a few floods in a year (Markus and Demissie, 2006).

Global climate change issue is one of the important topic for recent research, the results suggest that climatic instability will increase and resulting in greater frequency and intensity of extreme weather events (Senior *et al*., 2002). Beside floods, other potential consequence will be the acceleration erosion and sedimentation rates, responding to changes both in rainfall volume and intensity, with consequences for topsoil degradation, loss of agriculture productivity and increased export of sediment and contaminants especially from agriculture fields.

Problems requiring estimate of total runoff, erosion and sediment yield or statistical features of those variables, can be addressed by model for examples complex physically-based models. Recent physically event-based rainfall-sediment-runoff models include KINEROS (Smith *et al*., 1995), WEPP (Nearing, 1999), LISEM (De Roo *et. al*., 1996a,b), and EUROSEM (Morgan *et al*., 1998) has improved steadily of the prediction potential of soil loss rates from the average annual estimations using the Universal Soil Loss Equation approach (Wischmeier and Smith, 1978) to increasingly complex rainfall-sediment-runoff models capable of estimating the consequences of single rainfall events. Overall, and despite improved spatial and temporal discretization and better process description, simulation results are highly variable and very sensitive to input parameters (Jetten *et al*., 2003). This uncertainty can be related with variability in model input data, model structure, and with the non-linear nature of the rainfall-sediment- runoff processes.

Although model results can generally be improved by calibration, calibrating for future condition is not possible. Therefore, rainfall-sediment-runoff models for climate change analysis should be a robust model capable to reasonably perform with similar parameters values, including highly dynamic for the widest possible range of space-time conditions. Physically based distributed model-MEFIDIS was developed to assess the risk posed by unusually intense storm events for flooding and land degradation in medium-sized watersheds, in particular due to climate and land-use changes. The robustness of MEFIDIS for climate and land-use changes assessment was evaluated using a single parameter set, although this still increases the computational power needed to run the model (Nunos *et al*., 2005). It related to the problems that the fully distributed models still have limitation in application because these models require much computation time to conduct a simulation especially for large catchment size and long-term simulation.

One of alternative way to propose a robust model is by reducing the number of non-dimensional parameters to the dimensional parameters which having physical meaning. For example, physically-based rainfall-runoff-TOPKAPI approach is a comprehensive distributed-lumped approach. Theoretically it proves that the lumped version of the TOPKAPI model can be derived directly from the results of the distributed version and does not require additional calibration. At each time step in time the number of saturated cells is compared with the total volume of water stored in the catchment can be approximated by a beta distribution function curve, and finally the lumped TOPKAPI model parameters can be derivated from the distributed results. The parameter values of lumped TOPKAPI Model are shown to be scale independent and obtainable from distributed information of topographic, land-use, and soil type. The main advantage of this approach can be applied at increasing spatial scales without losing model and parameter physical interpretation. The model is allowed to be suitable for land-use and climate change impact assessment, its extension to ungauged catchments, for the extensive simulations, and coupling with General Circulation Models (GCMs) (Liu and Todini, 2002).

Another concept proposed lumping a distributed rainfall-runoff model using digital topographic information to reduce computational burden required in a long-term runoff simulation. The lumped model version was derived from relationship between storage volume and discharge over the entire catchment by considering spatial distribution of topographic variables and water content of watershed by assuming that a rainfall-runoff process of the system reaches a steady state by spatially uniform rainfall input. The lumped model basically gave good simulation results and required less computation time than the distributed model (Ichikawa and Shiiba, 2002).

This research article presents a method to lump a physically based distributed rainfall-sediment-runoff model as the tool for estimates spatio-temporal erosion, runoff, and sediment transport processes in order to reduce computation time and computer resources to conduct a long-term rainfall-sediment-runoff simulation in a large catchment or sub-catchment size. The equations and physical meaning of the lumped model parameters derived from distributed approach. The lumping method divided in two parts. First, lumping of physically based distributed rainfall- runoff model and second, lumping of physically based distributed rainfall-sediment-runoff model. As the extension from the lumping method of physically-based distributed rainfall-runoff model proposed by Ichikawa and Shiiba (2002), the authors have been adopted and extended that method with adding new lumping method for sediment transport processes. Because the dominant processes of erosion and sediment transport is effected only by surface flow, lumping of physically-based distributed rainfall -runoff model in initial stage of model development does not consider subsurface flow processes. To demonstrate the quality of the lumping physically-based distributed rainfall-sediment-runoff approach, the authors did numerical experiments for water and sediment discharges and compared the results with water and sediment discharges simulated by distributed model version. To investigate the performance of the distributed model and lumped model, these models applied in the Lesti River, Indonesia.

2. Physically-Based Distributed Rainfall-Sediment-Runoff Model

2.1 Cell Distributed Rainfall-Runoff Model

Since soil erosion and sediment transport by water is closely related to rainfall and runoff processes, erosion and sediment transport modelling cannot be separated from the procedures to use runoff generation model by using physically-based hydrological model. Cell Distributed Rainfall-Runoff Model Version 3 (CDRMV3) developed at Innovative Disaster Prevention Technology and Policy Research Laboratory DPRI-Kyoto University (Kojima *et al.*, 2003) including stage-discharge relationship with saturated-unsaturated flow (Tachikawa *et al.*, 2004) already applied for our study site. The model uses the Lax-Wendroff finite difference scheme to solves the one-dimensional kinematic wave equation with the stage-discharge equation to simulate runoff generation and routing. The simulation area is divided into an orthogonal matrix of square cells (250 m x 250 m), assumed to represent homogenous conditions based on DEM data.

The model assumes that the flow lines are parallel to the slope, the hydraulic gradient is equal to the slope. The kinematic wave of model does not consider the vertical water flow like infiltration effects. The input rainfall data *r(t)* is directly added to subsurface flow or surface flow according to the water depth on the area where the rainfall dropped.

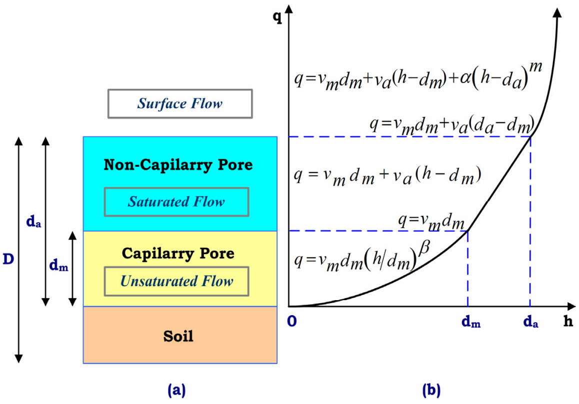


Fig. 1 (a) Concept of hillslope soil layer structure and (b) stage-discharge relationship of CDRMV3.

The model incorporating soil layer model structure which consists of a capillary pore which unsaturated flow occurs inside and a non-capillary pore in which saturated flow occurs. In the soil layer, slow flow and quick flow are modeled as unsaturated Darcy flow with variable hydraulic conductivity and saturated Darcy flow. According to this mechanism, surface flow will occurs if the water levels are higher than the total soil depth. In the CDRMV3, the horizontal sub-surface and surface flows, *q* (discharge with unit width), are calculated by the approximation equation (1) corresponding to stage-discharge relationship (Fig. 1) defined as:

$$q=\begin{cases} v_m d_m (h/d_m)^{\beta}, & 0\le h\le d_m \\ v_m d_m + v_a(h-d_m), & d_m\le h\le d_a \\ v_m d_m + v_a(h-d_m)+\alpha(h-d_a)^m, & d_a\le h \end{cases} \quad \text{........ (1)}$$

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=r(t) \quad \text{........ (2)}$$

The continuity equation (2) takes into account flow rate at each grid-cell or slope segment according governing equations at the above.

$$\text{With} \quad \begin{cases} v_m = k_m i \\ v_a = k_a i \\ k_m = k_a / \beta \\ \alpha = \sqrt{i} / n \end{cases}$$

Where i is the slope gradient, k_m is the saturated hydraulic conductivity of the capillary soil layer, k_a is the hydraulic conductivity of the non-capillary soil layer, d_m is the depth of the capillary soil layer, d_a is the soil depth, and n is the roughness coefficient based on the land cover classes.

The model parameters to be determined are n ($m^{-1/3}s$), k_a (m/s), d_a (m), d_m (m), and β. For river flow routing, surface flow with rectangular cross section is assumed for kinematic wave approximation.

2.2 Coupling of Cell Distributed Rainfall-Runoff Model and Sediment Transport Model

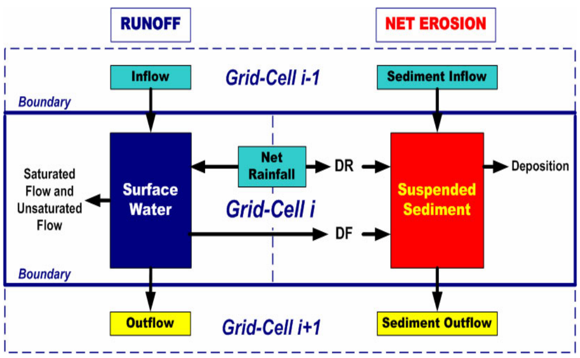


Fig. 2 Schematic diagram of the physically based rainfall-sediment-runoff model at a grid-cell scale.

Many distributed hydrological models have been developed and utilized, but there has been a few tries to couple these with sediment transport and sediment yield models. A physically-based distributed rainfall-sediment-runoff model deriving from the integration in space of the kinematic wave model was developed by authors, sediment transport algorithm added to the distributed rainfall-runoff model (CDRMV3) based on physical processes and investigated the characteristics of soil erosion based on field observation in our study area (Nakagawa *et al.*, 2005).

The basic assumption of this model is that the sediment is yielded when surface flow occurs which calculated based Kinematic Wave Runoff (KWR) model. The concept of spatially distributed modeling is shown in Fig. 2. The simulation area is divided into an orthogonal matrix of square cells, assumed to represent homogenous conditions which has the same resolution as DEM (250 m). Runoff generation, soil erosion and deposition are computed for each grid-cell.

Soil detachment and transport is handled with the continuity equation representing soil detachment by raindrop (*DR*) and surface flow (*DF*) ($kg/m^2/hr$) as follows :

$$\frac{\partial(h_s c)}{\partial t} + \frac{\partial(q_s c)}{\partial x} = e(x,t) \qquad \text{........ (3)}$$

With $e(x,t) = DR + DF$

Where C is the sediment concentration in the flow (kg/m^3), h_s is the water level of surface flow (m), q_s is the discharge of surface flow (m^3/s), e is the net erosion ($kg/m^2/hr$).

The net erosion is calculated by adding *DR*+*DF*. From observation of rainfall characteristic in our study area, the empirical equation for *DR* for each grid-cell expressed as a linear function (Oishi *et al.*, 2005) as follows :

$$DR_i = k\,KE = k\,56.48\ r_i \qquad \text{........ (4)}$$

Where k is the soil detachability (kg/J) (= 0.002), *KE* is the total kinetic energy of the net rainfall (J/m^2), and r is the depth of rainfall intensity (mm/h).

For each grid-cell, soil detachment and deposition by flow *DF* is simulated as a result of surface flow, following the sediment Transport Capacity of water (*TC*). Surface flow has limitation for transporting sediment at each grid-cell, therefore this study considers *TC* of surface flow at each grid-cell in each time step. If actual suspended sediment from upper grid-cells is lower than this capacity detachment occurs, otherwise excess soil deposition (Govers, 1990). The equation is :

$$DF_i = \alpha\,(TC_i - C_i)\,h_{si} \qquad \text{........ (5)}$$

Where α is the detachment/deposition efficiency factor, TC is the sediment transport capacity of overland flow (kg/m^3/h). $DR+DF$ are as defined for equation (3).

The transportation capacity is calculated by equation (7), which is based on the Unit Stream Power (USP) theory. The grid-cell based KWR model simulates the mean velocity v and the water depth h at each grid-cell. Sediment transport capacity is the product of the C_t and h. Yang (1972) derived a relationship between unit stream power and total sediment concentration as :

$$\log C_t = I + J \log((vi - v_{critical} i)/\omega) \quad \text{….. (6)}$$

$$TC = C_t \, h \quad \text{…… (7)}$$

In which

$$\omega = \sqrt{\frac{2}{3} + \frac{36}{DSTAR}} - \sqrt{\frac{36}{DSTAR}}$$

$$DSTAR = \left(\frac{\rho_s}{\rho_w} - 1\right) g \left(\frac{D50}{1000}\right)^{2/NU^2}$$

$$I = 5.0105$$
$$J = 1.363$$

Where C_t is the total sediment concentration (ppm), vi is the unit stream power (m/s), i is the slope gradient (m/m), $v_{critical}i$ is the critical unit stream power, $v_{critical}$ is the critical flow velocity, and ω is the sediment terminal fall velocity in water (m/s), ρ_s is the sediment particle density (kg/m^3), ρ_w is the water density (kg/m^3), g is the specific gravity (m/s^2), and $D50$ is the median of grain size (mm).

3. Physically-Based Lumped Rainfall-Sediment-Runoff Model

The main objective of the method to lump a physically-based distributed rainfall-sediment-runoff model is to derives equations and physical meaning of the physically-based lumped rainfall-sediment-runoff model parameters. The lumped model parameters are derived by analytically the integration of the distributed model.

The physically-based lumped rainfall-sediment-runoff model which has been developed by authors consists of one non-linear model for hydrological response and one linear model for sediment transport processes. We used the same concept with distributed model version to explore how sediment load is related to catchment hydrological response, detachment source type and intensity, and depositional processes. It is well known that hydrologically-driven detachment model must faithfully simulate surface responses. Therefore, the sediment-runoff processes in this study are affected by surface water, we have been developed a storage-type sediment-runoff.

For more detail, the lumping of distributed approach can be solved analytically with the following results:

3.1 Lumping of Physically-Based Distributed rainfall-Runoff Model

For lumping physically-based distributed rainfall-runoff model we used the method proposed by Ichikawa and Shiiba (2002). The most importance feature of this method is that the method does not presumptively give a bulk relationship between outflow discharges and storage volume at the catchment scale but it deductively derives a relationship from a physically-based approach by considering spatial distribution of topographic variables.

Based on the assumption of steady state conditions of rainfall-runoff by spatially uniform rainfall input, the relationship between total storage of water at the i^{th} grid-cell (s_{wi}) and the discharge flow at the i^{th} grid-cell (Q_{wi}) can be theoretically derived. Flux of Q_{wi} is expressed as the product of rainfall intensity ($\bar{r}$) and Upslope contributing areas (U). Upslope contributing area can be calculated from a DEM from each grid-cell. The storage of water at the catchment scale (S_w) can be calculated by adding up the s_{wi} from each grid-cell as a function of the topographic variables.

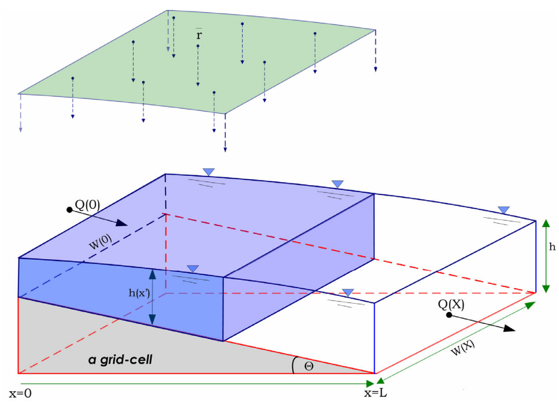


Fig. 3 Schematic diagram of the physically-based rainfall-runoff model at a grid-cell scale.

Flux of water discharge at a grid-cell $Q(x)$ (see Fig. 3) is expressed as the product of the rainfall intensity (spatially uniform) and the upslope contributing area as follows :

$$Q_i(x) = Q_i(0) + \bar{r}\int_0^x w_i(x)\ dx \qquad \text{........ (8)}$$

Where x is the horizontal distance from the upstream end of a grid-cell, L is total length of the horizontal distance of the grid-cell. The upslope contributing area can be generated from water flow accumulation data at each grid-cell based on DEM data. Because a grid-cell is an orthogonal matrix of square cell, $w(x)$ is constant in the unit. Then the equation (8) can be rewritten as :

$$Q_i(x) = Q_i(0) + \bar{r}\ x_i\ w(x)$$

$$= \bar{r}\, U_i + \bar{r}\ x_i\ w(x)$$

Flux of water discharge can be rewritten with unit width becomes :

$$Q(x)/w = \bar{r}\,U/w + \bar{r}\ x\ w(x)/w$$

$$q_i(x) = \bar{r}\,U_i/w + \bar{r}\ x_i$$

$$= \bar{r}\,(\frac{U_i}{w} + x_i) \qquad \text{......... (9)}$$

From stage-discharge relationship the general kinematic wave equation (q~h) for surface runoff as follows :

$$q(x) = \alpha\ h(x)^m \qquad \text{........ (10)}$$

In which

$$\alpha = \frac{\sqrt{i}}{n} = \frac{\sqrt{\sin\theta}}{n}$$

Where i is the slope gradient (m/m), n is the roughness coefficient, m is the exponent constant, which can be shown to be 5/3 from manning's equation.

Substituting equation (10) into (9), we obtain expression for surface water depth with the following result :

$$\alpha_i\ h_i(x)^m = \bar{r}\ (\frac{U_i}{w} + x_i)$$

$$h_i(x) = \left(\frac{\bar{r}\left(\frac{U_i}{w} + x_i\right)}{\alpha_i}\right)^{1/m} \qquad \text{........ (11)}$$

The storage volume of surface water for a grid-cell (s_{wi}) as the differential functions of water depth, the width of grid-cell and horizontal distance from upstream to downstream end is given as :

$$s_{wi} = \int_0^x h_i(x)\ w_i(x)\ dx$$

$$= w\int_0^L \left(\frac{\bar{r}\left(\frac{U_i}{w} + x_i\right)}{\alpha_i}\right)^{1/m} dx$$

$$= w\left(\frac{\bar{r}}{\alpha_i}\right)^{1/m} \frac{1}{\frac{1}{m}+1}\left[\left(x_i + \frac{U_i}{w}\right)^{\frac{1}{m}+1}\right]_0^L$$

$$= w\left(\frac{\bar{r}}{\alpha_i}\right)^{1/m} \frac{1}{\frac{1}{m}+1}\left(\left(L_i + \frac{U_i}{w}\right)^{\frac{1}{m}+1} - \left(\frac{U_i}{w}\right)^{\frac{1}{m}+1}\right)$$

$$s_{wi} = w\ \bar{r}^{\,p}\ k_i \frac{1}{p+1}\left(\left(L_i + \frac{U_i}{w}\right)^{p+1} - \left(\frac{U_i}{w}\right)^{p+1}\right)$$

…….. (12)

Where $p = \frac{1}{m}$, and $\left(\frac{1}{\alpha}\right)^{\frac{1}{m}} = k$

The total storage volume of surface water in the catchment system (S_w) is calculated by adding up the total storage volume from each grid-cell inside catchment as a function of the topographic variables and rainfall intensity :

$$S_w = \sum_{i=1}^{N} s_{wi}$$

$$= \sum_{i=1}^{N} w\ \bar{r}^{\,p}\ k_i \frac{1}{p+1}\left(\left(L_i + \frac{U_i}{w}\right)^{p+1} - \left(\frac{U_i}{w}\right)^{p+1}\right)$$

Where N is the total of the grid-cells.

Because we assume that the rainfall-runoff process reaches steady state with spatially uniform rainfall input, $\bar{r}$ can be expressed as functions of outlet discharge and total area of catchment system, then the above equation becomes :

$$S_w = \sum_{i=1}^{N} w \left(\frac{Q}{A}\right)^{p} k_i \frac{1}{p+1}\left(\left(L_i + \frac{U_i}{w}\right)^{p+1} - \left(\frac{U_i}{w}\right)^{p+1}\right)$$

…….. (13)

Finally, equation (13) describes a relationship between storage volume and outlet discharge of the catchment system obtained from the above procedures. Equation (13) can be simplified to expressing that relationship by using a parameter which is having physical meaning as follows :

$$S_w = K\,Q^p \qquad \text{…….. (14)}$$

K as the model parameter having a physical meaning, can be interpreted as the time of concentration for a kinematic wave to travel across the system. From equation (13), K is influenced by the length of path, its slope, roughness coefficient, upper contributing area, and total area. Its prove that K can be derived from the integration of distributed equation. Now K is dimensional parameter ($m^{6/5}s^{3/5}$) is defined as :

$$K = \sum_{i=1}^{N} \frac{w}{(A)^p} \frac{k_i}{p+1}\left(\left(L_i + \frac{U_i}{w}\right)^{p+1} - \left(\frac{U_i}{w}\right)^{p+1}\right)$$

…….. (15)

3.2 Lumping of Physically-Based Distributed rainfall-Sediment-Runoff Model

Sediment transport capacity at each grid-cell (TC_i) is function of topographic variables and hydrological responses. For each time step **s**ediment concentration **(**C) is assumed to be uniform over the catchment and this is the variable of sediment continuity (see Equation 21). The maximum sediment storage of the catchment ($S_s^{\max}$) can be calculated from each grid-cell based on s_{wi} and TC_i which are mathematically derived from S_w.

The Maximum Sediment Storage of the Catchment ($S_s^{\max}$)

The maximum sediment storage is defined as the total sediment transport capacity in a whole the catchment for each time step. Therefore, we expressed $S_s^{\max}$ as the function of sediment transport capacity and total storage of water.

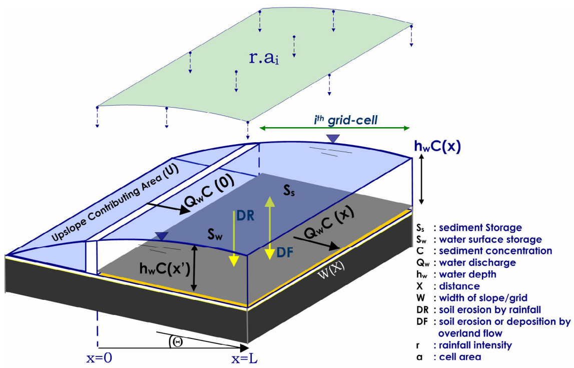


Fig. 4 Schematic diagram of the physically-based rainfall-sediment-runoff model at a grid-cell scale.

Physically, the maximum sediment storage can not be calculated by adding up the sediment transport capacity from each grid-cell (TC_i) directly, but need to multiply it with total storage water at each grid-cell (s_{wi}). The physically-based distributed rainfall-sediment-runoff approach can be simplified to derive physical meaning of the model parameter $S_s^{\max}$.

Based on the same assumption with the lumping method of physically-based distributed rainfall-runoff model, several variables of TC_i and s_{wi} can be calculated at each grid-cell (see Fig. 4).

From equation (9) and equation (11), v_i has been estimated by :

$$v_i\ (x) = \frac{q_i(x)}{h_i(x)}$$

$$= \frac{\bar{r}\left(\frac{U_i}{w} + x_i\right)}{\left(\frac{\bar{r}\left(\frac{U_i}{w} + x_i\right)}{\alpha_i}\right)^p}$$

$$= \frac{\bar{r}\left(x_i + \frac{U_i}{w}\right)}{\left(\bar{r}\left(x_i + \frac{U_i}{w}\right)\right)^p k_i}$$

$$= \left(\bar{r}\left(x_i + \frac{U_i}{w}\right)\right)^{1-p} k_i^{-1}$$

For a grid-cell $x = L$, then the above equation rewritten becomes :

$$v_i = \left(\frac{Q}{A}\left(L_i + \frac{U_i}{w}\right)\right)^{1-p} k_i^{-1} \qquad \text{…….. (16)}$$

Where v_i is independent variable of Unit Stream Power at each grid-cell (USP_i) as follows :

$$USP_i = v_i \sin\theta_i = v_i\ i_i$$

$$USP_i = \left(\frac{Q_w}{A}\left(L_i + \frac{U_i}{w}\right)\right)^{1-p} k_i^{-1}\ i_i$$

TC for a grid-cell (TC_i) has been estimated by :

$$TC_i = 10^{5.0105 + 1.363\log((USP_i - USP_{critical})/\omega)}$$

…….. (17)

The maximum sediment storage at the catchment scale is calculated by adding up TC_i multiply to s_{wi} for all grid-cells as follows :

$$S_s^{\max} = \sum TC_i\ s_{wi}$$

$$= \sum_{i=1}^{N} w\ \bar{r}^{\,p}\ k_i \frac{1}{p+1}\left(\left(L_i + \frac{U_i}{w}\right)^{p+1} - \left(\frac{U_i}{w}\right)^{p+1}\right) TC_i$$

$$= \sum_{i=1}^{N} w\left(\frac{Q}{A}\right)^p k_i \frac{1}{p+1}\left(\left(L_i + \frac{U_i}{w}\right)^{p+1} - \left(\frac{U_i}{w}\right)^{p+1}\right) TC_i$$

…….. (18)

Sediment Concentration (C) :

The relationship between detachment and redeposition represented by equation (5) depends to the balance between the actual sediment storage per unit area (S_s) (kg/m²/h) and the maximum sediment storage per unit area ($S_s^{\max}$) (kg/m²/h). Based on the relation between S_s and S_w, the value of C from watershed outlet for each time can be solved as follows :

$$C(t) = \frac{S_s(t)}{S_w(t)} \qquad \text{……. (19)}$$

As a first stage of the development, neglect subsurface contributions, and assume that all the water flows as surface flow.

Finally, the continuity equations of S_w and S_s on a catchment scale are presented as follows :

$$\frac{dS_w}{dt} = \bar{r}\ A - \left(\frac{S_w}{K}\right)^{1/p} \qquad \text{….. (20)}$$

$$\frac{dS_s}{dt} = DR + DF - Q_w C$$

$$= k\ 56.48\,\bar{r} + \left(\alpha\left(S_s^{\max} - S_s\right) - Q_w C\right)/A$$

….. (21)

Where A is the total area of the catchment (m^2), r is the actual rainfall intensity (m/hr), K is the catchment response as the model parameter, α is the detachment/deposition efficiency factor, and k is the soil detachability (kg/J).

Runoff and sediment load calculations are conducted by using those parameters K and S_s^{max} obtained from the above procedures and continuity equations. The continuity equations of water and sediment were solved numerically using Rungge-Kutta method order 4th .

The most important feature that the model has two kind model parameters which having physical meaning, they are K and S_s^{max}. The physical meaning of the lumped model parameters K and S_s^{max} are resulted from the lumping method of physically-based distributed rainfall-sediment-runoff model. Model parameter K derived from the lumping method of physically-based distributed rainfall-runoff model (CDRMV3), and model parameter S_s^{max} can be derived from coupled of CDRMV3 and sediment transport algorithm.

4. Application of the Physically-based Distributed Rainfall-Sediment-Runoff Model

The hydrological dataset in the year 2003 is selected as an example for the verification of the coupled CDRMV3 and sediment transport algorithm in the Lesti River catchment. Since in that year from 3 to 6 October, 2003, a relatively large flood event with a peak discharge of 54 m^3/s occurred. Hourly measurements from three automatic raingauges and one turbidity-discharge at the watershed outlet were available. The areal rainfall distribution was estimated by the Thiessen Polygon method. Sub-surface and surface flow by using equation (1) was incorporated.

The model calibration was performed at 1-hour time step for the hydrological dataset from 3 to 6 October, 2003 with the model parameter set is given in Table 1.

Table 1. Physically-based distributed rainfall-runoff-sediment model parameters used in the Lesti River (3 to 6 October, 2003)

Symbol	Description
d_m	The depth of the capillary soil layer (m) $d_m = 0.8$
d_a	The depth of capillary soil layer + non-capillary soil layer (m) $d_a = 1.2$
k_a	The hydraulic conductivity of the non-capillary soil layer (m/s) $k_a = 0.008$
β	The non-linear exponent constant for the capillary soil layer $\beta = 30.0$
n	The roughness coefficient based on the land cover classes ($m^{-1/3}s$) The range of n values 0.03-3
D_{50}	The median grain size (mm) $D_{50} = 0.062$
k	The soil detachability (kg/J) $K = 0.001$
α	The detachment or deposition efficiency $\alpha = 0.98$
ρ_s	The sediment particle density (kg/m^3) $\rho_s = 2650$
ρ_w	The water density (kg/m^3) $\rho_w = 1000$
g	The specific gravity (m/s^2) $g = 9.8$
NU	The kinetic viscosity of water (m^2/s) $NU = 10^{-6}$

4.1 The Study Area

The Lesti River catchment is a small catchment (351.3 km^2) inside the Brantas River basin (11,800 km^2), east Java in Indonesia. The Lesti River catchment is located at the upstream of the Brantas River basin. The Lesti River catchment is also instrumented for rainfall, runoff, erosion, and sediment yield measurement. This area represents a tropic-volcanic area covered with forest, agriculture, urban, and shrub vegetation. The Lesti River catchment is mostly covered with volcanic soil originated from Mt. Semeru, an active volcano located at the upstream of the Lesti River. At the merging point of the Lesti River and the Brantas main reach, Sengguruh dam was constructed in 1998 for water resources and power generation. Unexpectedly, most of the gross storage (21.5 million m^3) has been already filled with the large amount of sedimentation from the Lesti River and the main reach of the Brantas River.

4.2 Simulation Result

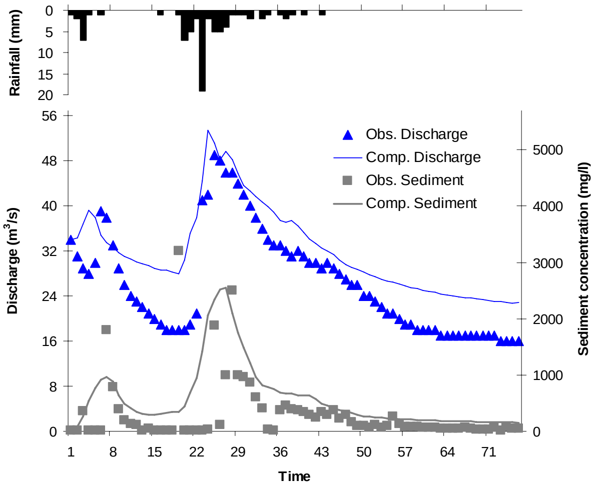


Fig. 5 Comparison of predicted runoff event and sediment discharge with observed data for 3 to 6 October 2003.

One of the simulated flood events is shown in Fig. 5. The predicted runoff and sediment concentration were compared with the observed one. A summary of this flood runoff event is given by the model efficiencies are : 0.854 (R^2 : the coefficient of determination), 0.251 (*RRMSE* : Relative Root Mean Square Error), and 5.831 (absolute error) (see Appendix-1). We may be able to recognize that the model prediction from an example is closer to the observed value.

4.3 Spatial Patterns of Erosion and Deposition

Fig. 6 shows simulated erosion and deposition areas inside watershed for flood runoff event 3 to 6 October 2003. A direct comparison with observed data is difficult since the threshold erosion rate above which these areas are mapped is not known. Fig. 6 shows that the model was capable of locating the main sediment sources and sink within the Lesti River during this flood event. The model was also able to assign major erosion features.

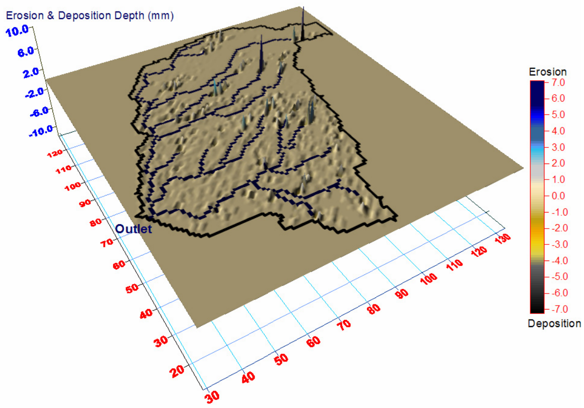


Fig. 6 Erosion and deposition sources distribution in watershed for 3 to 6 October 2003.

5 Evaluation of The Physically-based Lumped Rainfall-Sediment-Runoff Model

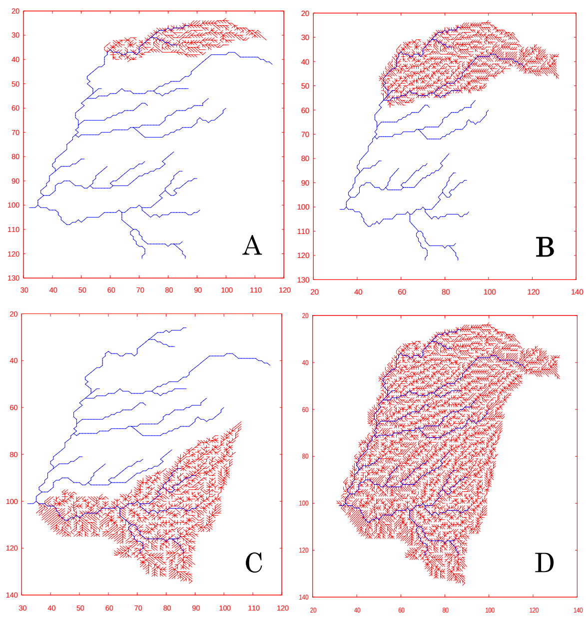


Fig. 7 Lesti River catchment as partioned into four catchment levels based on the upscalling area where red lines are the catchment area and blue lines are channels river.

As explained previously, the lumped rainfall-sediment-runoff model parameters can be derived directly from the distributed approach of

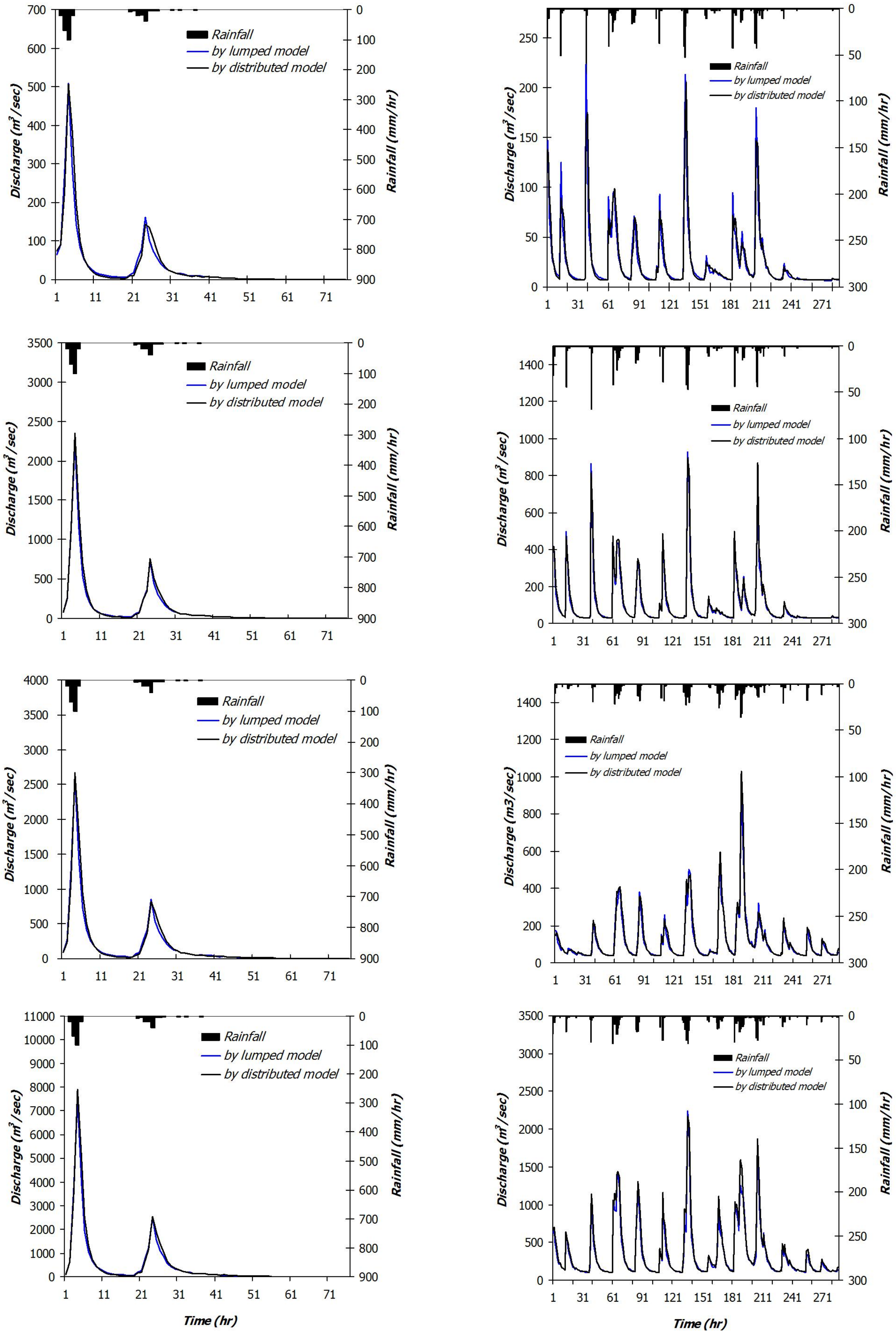


Fig. 8　Comparison of calculated water discharges using the distributed and lumped models for two periods at different catchment sizes (A,B,C and D) inside the Lesti River catchment.

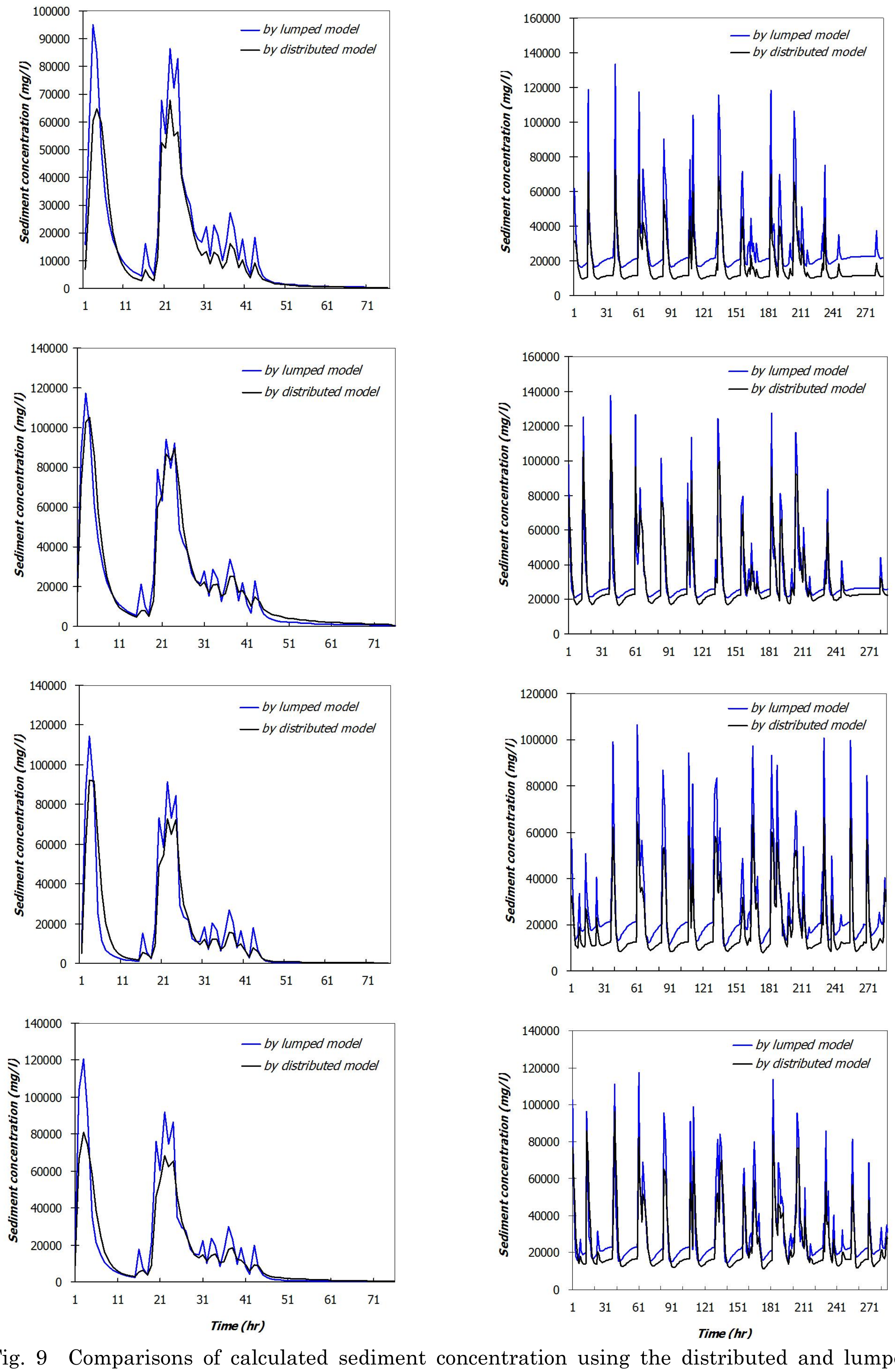


Fig. 9 Comparisons of calculated sediment concentration using the distributed and lumped models for two periods at different catchment sizes (A,B,C and D) inside the Lesti River catchment.

physically-based distributed rainfall-sediment-runoff model version. For simplification, the distributed model neglects the sub-surface flow processes for this lumping experiment. The model which was lumped was applied to Lesti River catchment and we evaluate the performance of the lumped model. The numerical experiments was run for two cases of hourly rainfall data input (case 1 : short duration, and case 2 : long duration) and four cases catchment size. Four scenarios of the catchment size were delineated (see Fig. 7) and its characteristics are given in Table 2.

Table 2. The characteristics of four cases catchment size in the Lesti River catchment (Fig. 7)

Catchment	**Total grid-cell**	**Area (km^2)**
Case A	404	25.3
Case B	1743	108.9
Case C	2114	132.1
Case D	6101	351.3

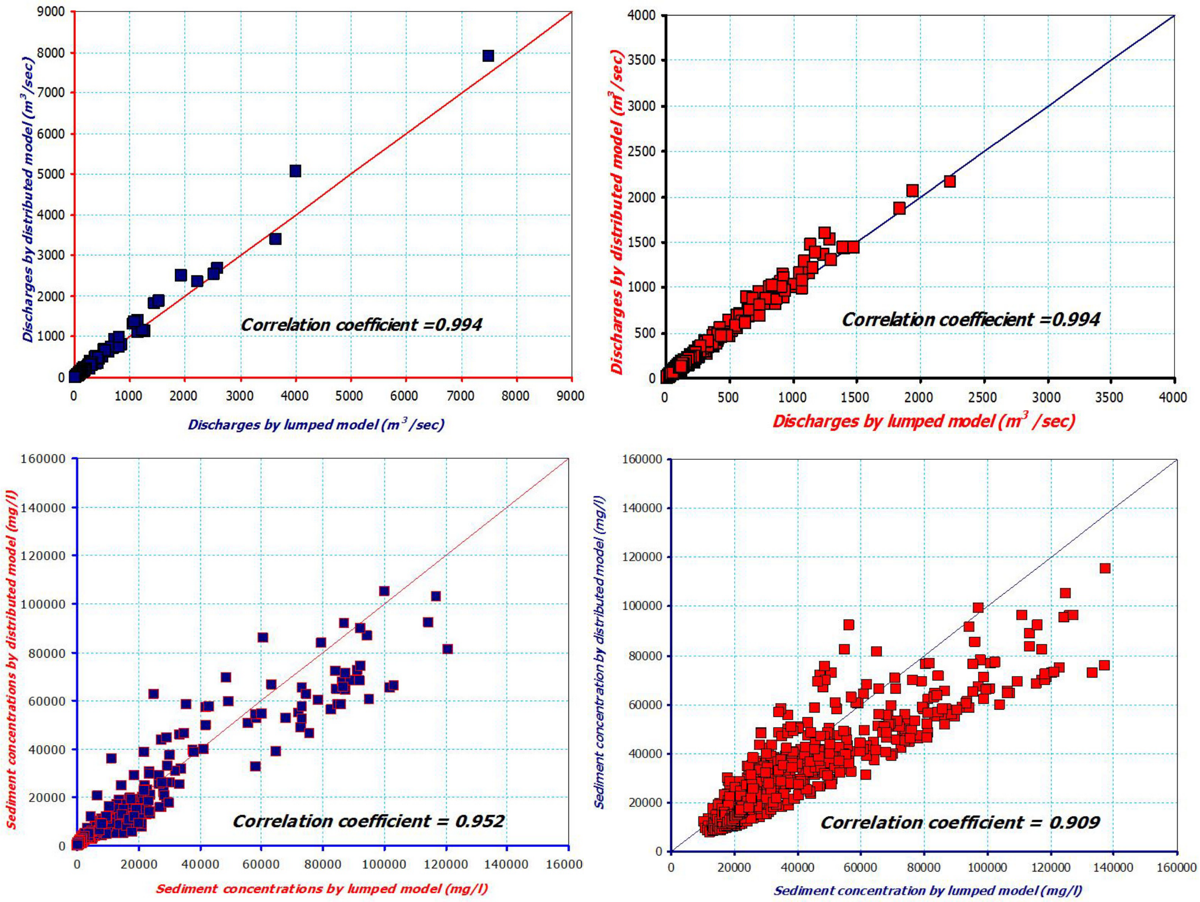
A digital topographic model for the Lesti River catchment was first developed and the area divided into four catchment areas defined based on the four scenarios of catchment size and topographic model. Then the lumped model developed in the previous section was applied to a catchment area of each scenario. The parameter values of the lumped grid-cell model were as follows : n = 0.2 m$^{-1/3}$s except for river channel n = 0.001 m$^{-1/3}$s, m = 1.667, d_m = 0 m, d_a = 0 m, D_{50} = 0.062 mm, k = 0.004 kg/J, α = 0.98, ρ_s = 2650 kg/m^3, ρ_w = 1000 kg/m^3, g = 9.8 m/s^2, and NU = 10^{-6} m^2/s.

Fig. 8 and Fig. 9 show comparisons of simulated water and sediment discharges (hydrograph and sedimentgraph) for two periods rainfall data input and four cases catchment size which calculated by the distributed and the lumped model version. The black lines are simulated water and sediment concentration by the distributed model and blue lines are simulated water and sediment concentration by the lumped model.

Fig. 8 shows only marginal differences between the simulated discharges for the whole catchment size criteria and two kinds rainfall data input, generated by the physically-based distributed rainfall-runoff model and the lumped model. These cases demonstrate that the lumped version of the rainfall-runoff model is already suitable for reproducing the overall water discharge that reaches the river network generated by the distributed model. These results enhance the lumping method proposed by Ichikawa and Shiiba (2002), the lumped model which derived from distributed model basically gave good simulation results and required less computation time than the distributed model.

Fig. 9 plots sediment concentration by the physically-based distributed rainfall-sediment-runoff model and the lumped model. The outputs lumped model show overestimated sediment concentration for all simulation results to the distributed model results especially for peak times and low flows, any way the sediment concentration predictions of time when a sedimentgraph rises or recesses are good.

The lumped model performance qualitative and statistic for outflow discharges and sediment discharges based on two periods rainfall data input called case 1 and case 2 combined with all scenarios of catchment size given in Fig. 10. That figure shows that for both cases, the simulation results for outflow discharge and sediment discharge computed by lumped model basically approximated the simulation results computed by distributed model in the spatial and temporal changes. All the values of correlation coefficient more than 0.90 (see Appendix-2), its mean that the lumped model is in an acceptable way to reproduce the discharge and sediment discharge which calculated by distributed model. Just as with the case of sediment discharges the correlation coefficients lower than outflow discharges, the most of sediment discharges are overestimated for the smaller and larger simulated values calculated by distributed model. On the other hand, in the case of rainfall data input where total rainfall accumulation in case 2 is higher than case 1, the error of the case 2 is larger than case 1.



(a) (b)

Fig. 10 Comparison of simulated outflow discharges and sediment discharges by lumped model and distributed model for two periods rainfall data input : case 1 (a) and case 2 (b). The solid line is 1 to 1.

6 Summary

The lumped rainfall-sediment-runoff model is developed by lumping physically-based distributed rainfall-sediment-runoff model without introducing any tuning parameter. The distributed model is used to identify the mechanism governing the dynamics of the surface runoff as a function of the total water storage, and sediment concentration as functions of the sediment transport capacity of surface flow.

Lumping of physically-based distributed rainfall-sediment-runoff model which based on steady state condition and spatially uniform rainfall data basically gave good simulation results. Discharges simulated by the lumped model agree well with discharges simulated by distributed model. The sediment concentration predictions of time when a sedimentgraph rises or recesses are as well fitted to distributed model. The model runs efficiently in terms of calibrating and running time.

The main advantage of lumping distributed approach lies in it capability of being applied at large scales without losing model and parameter physical interpretation. By incorporating hydrological and sediment transport processes derived from the distributed model it is possible to reproduce the rainfall-sediment-runoff mechanism at catchment scale preserving physical values of the parameters.

The lumping physically-based distributed model is foreseen to be suitable for large catchments, land-use and climate change impact assessment, for extreme flood analysis, for use with General Circulation Models (GCMs).

Sub-surface processes as path of hydrological pathway has still to be introduced ; this was ignored in the initial stage since it was not dominant source in the sediment transport processes. This objective may be pursued through the introduction the lumping method of physically-based distributed rainfall-sediment-runoff model by incorporating sub-surface flow and river channel erosion and deposition. In addition, an approach for incorporating reservoirs and lakes should be included in further study.

References

De Roo, A.P.J., Wesseling, C.G., and Ritsema, C.J. (1996a): LISEM: A single event physically hydrological and soil erosion model for drainage basins: I-theory, input, and output. Hydrol. Process. 10(8), 1107-1117.

De Roo, A.P.J., Offermans, R.J.E., and Cremers, N.H.T.D. (1996b): LISEM: A single event physically hydrological and soil erosion model for drainage basins: II-sensitivity analysis, validation and application. Hydrol. Process. 10(8), 1119-1126.

Govers, G.R. (1990): Empirical relationships for the transporting capacity of overland flow. Int. Assoc. Hydrol. Sci. Publ. 189, 45-63.

Ichikawa, Y., and Shiiba, M. (2002): Lumping of kinematic wave equation considering field capacity. Third international conference on water resources and environmental research. 22nd-25th of July 2002 at Dresden University of Technology.

Jetten, V., Govers, G., and Hessel, R. (2003): Erosion models: quality of spatial predictions. Hydrol. Process. 17, 887-900.

Kojima, T., and Takara, K. (2003): A grid-cell based distributed flood runoff model and its performance, weather radar information and distributed hydrological modelling, IAHS Publ. No. 282, pp 234-240.

Liu, Z., and Todini, E. (2002): Towards a comprehensive physically-based rainfall-runoff model. Hydrology and Earth System Sciences, 6(5), 859-881.

Markus, M., and Demissie, M. (2006): Predictibility of annual sediment loads based on flood events. Journal of Hydrologic Engineering. 1084-0699, 354-361

Morgan, R.P.C., Quinton, J.N., Smith, R.E., Govers, G., Poesen, J.W.A., Chisci, G., and Torri, D. (1998): The EUROSEM model. In: Boardman, J., Favis-Mortlock, D.T. (Eds.), Modelling Soil Erosion by Water, NATO-ASI Series I-55. Springer, Berlin, pp. 389-398.

Nakagawa, H., Satofuku, Y., Muto, Y., Oishi, S., Sayama, T., and Takara, K. : On sediment yield and transport in the Lesti River basin, a tributary of the Brantas River, Indonesia-Experiences from field observations and remotely sensed data -, Proc. of International Symposium on Fluvial and Coastal Disasters, -Coping with Extreme Events and Regional Diversity-, Kyoto, Japan, 1 - 2 December 2005, CD-ROM, 2005.

Nearing, M.A., Govers, G., and Norton, L.D. (1999): Variability in soil erosion data from replicated plots. Soil Sci. Soc. Am. J. 63, 1829-1835.

Nunos, J.P., Viera, G.N., Seixas, J., Goncalvas, P., and Carvalhais, N. (2005): Evaluating the MEFEDIS model for runoff and soil erosion prediction during rainfall events. Catena. 61, 210-228.

Senior, C.A., Jones, R.G., Lowe, J.A., Durman, C.F., and Hudson, D. (2002): Predictions of extreme precipitation and sea-level rise under climate change. Philos. Trans. R. Soc. Lond. Ser. A: Math. Phys. Sci. 360 (1796), 1301-1311.

Smith, R.E., Goodrich, D.C., Woolhiser, D.A., and Unkrich, C.L. (1995): KINEROS-a kinematic runoff and erosion model. In: Singh, V.J. (Ed.), Computer Models of Watershed Hydrology, Water Resources Publications, pp. 697-732.

Tachikawa, Y., Nagatani, G., and Takara, K. (2004): Development of stage-discharge relationship equation incorporating saturated – unsaturated flow mechanism, Annual Journal of Hydraulic Engineering, JSCE, 48, 7-12.

Oishi, S., Sayama, T., Nakagawa, H. Satofuka, Y., Muto, Y., Sisinggih, D. and Sunada, K. (2005): Development of estimation method for impact energy of raindrop considering raindrop size distribution and the relationship between the impact energy and local sediment yield, Annual

Journal of Hydraulic Engineering, JSCE, vol. 49, pp. 1087 - 1092.

Wischmeier, W.H., and Smith, D.D. (1978): Predicting rainfall erosion losses, a guide to conservation planning. Agriculture Handbook, vol. 282. United States Department of Agriculture.

Yang, C. T. (1972): Unit stream power and sediment transport. J. Hydraul. Div. Am. Soc. Civ. Eng., 98(HY10), 1805-1826.

Appendix-1

Statistical indexes :

Model Efficiency (EF) :

$$EF = \frac{\sum_{i=1}^{n}\left(O_i - \overline{O}\right)^2 - \left(P_i - \overline{P}\right)^2}{\sum_{i=1}^{n}\left(O_i - \overline{O}\right)^2} \rightarrow -\infty < EF \leq 1$$

Relative Root Mean Square Error (RRMSE) :

$$RRMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(P_i - O_i\right)^2}{n}} \cdot \frac{1}{\overline{O}} \rightarrow 0 \leq RRMSE$$

Absolute Error (ABSERR) :

$$ABSERR = \frac{\sum_{i=1}^{n}\left|O_i - P_i\right|}{n} \rightarrow 0 \leq ABSERR$$

Where, P_i : predicted discharge

O_i : observed discharge

n : Number of P or O values

$\overline{P}$ and $\overline{O}$: the mean of the predicted and observed values over the time period

Appendix-2

Correlation coefficient :

$$R = \frac{\sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sqrt{\sum\left(X_i - \overline{X}\right)^2 \sum\left(Y - \overline{Y}\right)^2}}$$

Where, X_i : predicted value by lumped model

Y_i : predicted value by distributed model

$\overline{X}$ and $\overline{Y}$: the mean of the predicted values by lumped model and distributed model

物理型分布モデルの集中化による降雨土砂流出現象の時空間予測

ＡＰＩＰ∗・佐山敬洋・立川康人∗・寶　馨

∗ 京都大学工学研究科都市環境工学専攻

要　旨

分布型降雨土砂流出モデルの集中化手法を提案する。この方法は，キネマティックウェーブ式にもとづく分布型降雨流出モデルの集中化と，土砂生産・堆積過程を再現する分布型土砂流出モデルの集中化の二つのステップからなる。まず，流出現象の定常性を仮定することにより，流域全体の貯留量と流域下端からの流量との関係を導出する。次に，流域全体の土砂輸送可能量と水の貯留量との関係を，分布型土砂流出モデルの式から解析的に導出する。各時間ステップで状態量として計算する土砂濃度・貯留量と，土砂輸送可能量とのバランスにもとづき，土砂生産・堆積量を時々刻々推定する。提案する集中型モデルを，インドネシアのレスティ川流域に適用し，分布型降雨土砂流出モデルと比較することによって，その性能を検証する。

キーワード：集中化，分布型降雨土砂流出モデル，土壌侵食，レスティ川流域