

## Stochastic Operation for Multi-purpose Reservoir Using Neuro-Fuzzy Systems

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### Synopsis

We introduce a new approach for system optimization, named stochastic fuzzy neural network, which can be defined as a neurofuzzy system that is stochastically trained and can yield a 'quasi' optimal solution. The method intends to overcome some of the problems related to stochastic dynamic programming models. For validation, a real case of storage reservoir optimization with stochastic inflow discharge is considered. The SFNN model proved to show improvements in the optimization results. Moreover, to deal with the uncertainties related to discretization of inflows in finding the stochastic representative inflows, conditional probability of a fuzzy event is used.

**Keywords:** Stochastic optimization, Fuzzy Neural Network, Conditional probability of a fuzzy event, Reservoir operation

### 1. Introduction

One of the most important and applied techniques for the optimization of reservoirs is surely dynamic programming (DP), which was developed by Richard Bellman (1959). Applications of dynamic programming to water resources systems can be found in many works, such as Yakowitz (1982) and Esogbue (1989). DP presents various advantages over other methods to handle water resources management and optimization problems and it can be associated with other programming methods, such as stochastic techniques resulting in the so called stochastic DP (SDP). Some applications of SDP for water resources management is found in Torabi *et al.* (1973) and Kelman *et al.* (1990), where river discharge is treated as the stochastic variable. Moreover, DP may also be used for the optimization of multistage fuzzy control systems, called fuzzy DP (Fontane *et al.* 1997, Kojiri *et al.* 1993 and Hori *et al.* 1997), in which decision and state variables as well as

constraints, can be set as fuzzy membership functions. The basic characteristics of water resources and reservoir operation that lead to the use of DP are: stage-wise structure and non-linearity of the system.

DP can be divided into two different approaches as continuous or discrete, with the latter the most commonly used for computational convenience. However, for reservoir operation, the rough discretization of state variables, such as storage levels in the case of reservoir optimization, may reduce the efficiency of the optimization model. Another important characteristic of DP is the direction in which the calculation is carried out, defined as forward-moving and backward-moving. For deterministic DP (DDP), calculation in either direction can be easily implemented. However, the application of forward-moving calculation for SDP models is not straightforward and may be extremely complex to model (Chaves *et al.*, 2003). The direction in which the calculation is executed becomes extremely important when DP based optimization

models have to be combined or embedded with other models, such as rainfall-runoff and water quality simulation models (Chaves, 2002 and Chaves *et al.*, 2003), which require a time-wise calculation as actual values depend on previous ones. Moreover, the most important drawback of using DP is certainly the curse of dimensionality, caused by the large number of state and decision variables.

To overcome these problems related to SDP, a new optimization method is proposed here to handle the discretization of state variables through fuzzy theory and the limitations of the direction of calculation through a neuro-fuzzy system. The new proposed method is also able to handle the stochastic characteristics of variables, such as river discharge, in a forward-moving calculation direction. Moreover, the problem resulting from the increase in the number of state variables is easily overcome, as new state variables can simply be introduced as new neuron units. The proposed method is named stochastic fuzzy neural network (SFNN). For validation of the proposed method, a real storage reservoir optimization problem considering the stochastic characteristics of inflow discharge information is presented. Then, results using the SFNN are compared to the ones obtained by other dynamic programming formulations.

## 2. Methodology

There are few applications of ANN models for optimization problems, whereas the ANNs are usually applied as simulation or prediction models. It was only recently that some attempts to solve the storage reservoir optimization using ANN have appeared. Raman and Chandramouli (1996) and Chandramouli and Raman (2001) proposed the use of ANN to generate operating rules based on the optimal results from a deterministic DP model, for the case of a single and multiple reservoir system, respectively. In a similar way, Cancellier *et al.* (2002) developed a model to derive the operating rules for an irrigation supply reservoir. In an attempt to consider the stochastic characteristics of inflow, Ponnambalam *et al.* (2003) trained an adaptive neuro-fuzzy inference system (ANFIS; Jang, 1993) based on the optimal results obtained by a stochastic optimization model. Chang *et al.* (2001) combines genetic algorithm (GA) and ANFIS, in which GA is applied to search the optimal reservoir operating

histogram, which is used as the training pattern of the ANFIS model, intended to estimate the optimal water release with current storage level and inflow conditions as input information.

There are some important characteristics of the proposed method, which differ from other applications of neuro-fuzzy systems for reservoir operation. First, most of the ANN based models has the ANN trained based on already-optimized results from other models to derived operating rules. However, for the proposed SFNN model, the optimization is carried out directly considering the objective functions. The second and most important point is that the SFNN is trained directly under stochastic input conditions, presenting a probability of occurrence depending on the known previous value, defined as conditional probability. The most common approach to define the conditional probability is the Markov chain technique (Howard, 1960) which considers one previous stage of the conditional probability. The Markov chain is applicable for continuous processes or, for computational convenience, discrete processes. Some applications of optimization problems using the Markov chain technique can be found in Butcher (1971) and Yakowitz (1982). The inflow into the reservoir can be considered as a Markov chain process, where the system state, within a certain stage, is considered to be dependent on the state of the previous stage and the known probabilities.

Depending on the available data and the number of assumed intervals to pursue the calculation of the Markov chain or one previous stage conditional probability (CP), the resulting probabilistic curves may present unrealistic characteristics due to incomplete data set. The use of such results in stochastic models may greatly affect its efficiency. To reduce the uncertainties of the discretization process when calculating the conditional probabilities, due to incomplete data set and rough assumption of crisp discrete intervals, the introduction of conditional probability of a fuzzy event is proposed for the calculation of the conditional probabilities of the representative inflow discharges between two consecutive months.

## 3. Conditional Probability of an event

For the (nonfuzzy) conditional probability of a fuzzy event, the difference is that instead of crisp discrete intervals, the domain is divided into fuzzy linguistic variables as can be seen in the lower part of Fig 1, where

$L$ ,  $M$  and  $H$  stand for *low*, *medium* and *high*,  $V$  and  $X$  for *very* and *extreme*, respectively, with the center values of each linguistic variable described in the axis of the bottom part of Fig 1.

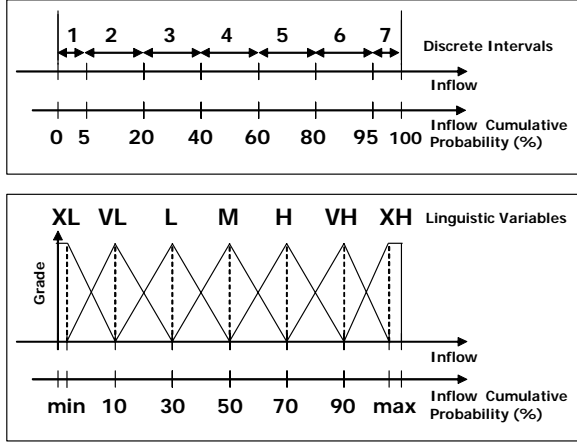


Fig 1. Discretization of discharge: discrete (upper) and fuzzy (lower) intervals

The percentage scale in Fig 1 represents the chosen limits for the discrete intervals and center values of linguistic variables, corresponding to the monthly inflow (unconditioned) cumulative probabilities. This increases the efficiency of the stochastic optimization model as already carried out by Chaves (2002), reducing to a certain extent null-probability values, as a result of many discretized interval or small time-series data.

Conditional probability is traditionally defined as the probability of an event  $A$  under the condition that an event  $B$  occurs. This probability is denoted by  $P(A/B)$  and defined as shown in equation ( 1 ).

$$P(A/B) = \frac{P(AB)}{P(B)} = \frac{P(A \cap B)}{P(B)} \quad (1)$$

for  $P(B) > 0$  and where  $P(AB)$  is the intersection of events  $A$  and  $B$ , and  $P(B)$  is the probability of event  $B$ . As stated by Kaufman (1975) and Kacprzyk (1997), the above concept is valid for both cases of nonfuzzy and fuzzy events.

For the case of the conditional probability of inflow discharge, we are interested to find the probability of inflow  $t$  when inflow  $t-1$  occurs, which would correspond to events  $A$  and  $B$ , respectively. The  $P(AB)$  can be calculated based on the observed time series data. For the case of discrete intervals, the frequency of occurrence  $F$  can be found through equation ( 2 ) as shown below:

if and only if  $(I_t \in A_k)$  when  $(I_{t-1} \in B_j)$  then :

$$F(A_k / B_j) = \sum_{n=1; c_0=0}^N c_{n-1} + 1 \quad (2)$$

Where  $c_n$  is a counter operator for which the term +1 accounts for each occurrence between consecutive events (intervals)  $A_k$  and  $B_j$ , whereas  $k$  and  $j$  are indices representing the intervals [1,2,...7] in the upper part of Fig 1. Note that the summation of all  $F(A_k/B_j)$  is equal to the total number of pairs of observations  $N$ .

$$P(A_k \cap B_j) = \frac{F(A_k / B_j)}{N} \quad (3)$$

Finally, as the probability  $P(B_j)$  of the inflow event  $B_j$  corresponding to interval  $j$  can easily be found by dividing the frequency of inflows contained within interval  $j$  by the total number of observations  $N$ . Then, it is possible to obtain the final one-previous-stage or Markov chain conditional probability using equation ( 4 ):

$$P(A_k / B_j) = \frac{P(A_k \cap B_j)}{P(B_j)} \quad (4)$$

For the case using the fuzzy method, the discharge values are fuzzified using the membership functions as shown in the lower part of Fig 1. Then, also based on the observed data, frequency of conditioned occurrence of a fuzzy event, hereafter denoted by  $F\mu(A_k/B_j)$  can be analogously calculated for each linguistic variable using equation ( 5 ):

$$F\mu(A_k / B_j) = \sum_{n=1}^N \mu_{A,k}(I_{t,n}) \cdot \mu_{B,j}(I_{t-1,n}) \quad (5)$$

Where  $A$  and  $B$  are now fuzzy consecutive events, for example inflows of January and February;  $\mu_{A,k}$  and  $\mu_{B,j}$  are the fuzzy membership function of these events for the linguistic value  $k$  and  $j$ , respectively,  $k$  and  $j$ : [XL,VL...XH], for each pair  $n$  of observed data (for example, for every year). The membership function here represents the degree to which an observed value corresponds to a certain class or linguistic variable. For example, if both consecutive values are equal 1, this would indicate that the observed values are equal to the representative inflows and equation ( 5 ) would yield the same results of frequency for the nonfuzzy method, equation ( 2 ). Values different than 1, could be roughly said to represent a certain degree of "frequency" for two consecutive observed values. Note that the summation of all  $F\mu(A_k/B_j)$  for  $k$  and  $j$ , is equal to total number of pair of observations  $N$ , analogous to the nonfuzzy counterpart

method. Hence, the  $P\mu(AB)$  of the fuzzy events can be then calculated using equation ( 6 ):

$$P\mu(A_k \cap B_j) = \frac{F\mu(A_k / B_j)}{N} \quad (6)$$

The (nonfuzzy) probability of a fuzzy event A is denoted by  $P\mu(A)$  and is defined as in equation ( 7 ), as described by Kaufman (1975) and Kacprzyk (1997), based on the classic definition of Zadeh (1968), which is by far the most popular and widely used.

$$P\mu(B) = \sum_{n=1}^N \mu_B(x_n) \cdot p(x_n) \quad (7)$$

Where  $x$  is the actual value of the variable in question for occurrence  $n$ . Hence, for our case of inflow discharge:

$$P\mu(B_j) = \sum_{n=1}^N \mu_{B,j}(I_{t-1,n}) \cdot p(I_{t-1,n}) \quad (8)$$

Where  $p(I_{t-1,n})$  is the probability of a certain inflow  $I_{t-1}$  and  $\mu_{B,j}(I_{t-1,n})$  is the value of the fuzzy membership function for the inflow corresponding to observation  $n$  for event (linguistic variable)  $j$ . That is to say,  $P\mu(B_j)$  is the probability of a fuzzy event  $B$  to occur. Note that the summation of  $p(I_{t-1,n})$  is equal to 1 and (in our case) for each pair of observation  $n$ , the probability of a certain observed inflow presents the same value  $1/N$ . Finally the conditional probability of a fuzzy event  $A_k$ ,  $P\mu(A_k/B_j)$ , is calculated according to equation ( 9 ).

$$P\mu(A_k / B_j) = \frac{P\mu(A_k \cap B_j)}{P\mu(B_j)} \quad (9)$$

The summation of  $P\mu(A_k/B_j)$  in respect to  $j$  is also equal to 1 (one). Note that the summation is equal to 1 only if the linguistic variables are arranged in a way such as illustrated in Fig 1, where the minimum values of membership functions for each class coincide with the center values of adjacent classes. In any other case, for example using a different membership function, the probability of each month has to be compensated by the total summation for all classes.

It is important to mention that the proposed method to define the Markov chain conditional probabilities is not dealing with the vagueness and uncertainties related to the probabilities themselves, but only the uncertainties originating from the discretization process and the fuzziness of discharge observations. For more sophisticated analyses to handle such probabilistic problems, the readers are referred to other methods such as possibility theory, Dempster-Shafer theory of

evidence and fuzzy measures.

#### 4. Stochastic Fuzzy Neural Network (SFNN)

Fig 2 shows the flowchart of the overall calculation procedures for the SFNN optimization model where inflow values are considered as stochastic variables based on conditional probabilities. According to Fig 2, first the SFNN model is initiated with a set of parameters, which can be randomly created.

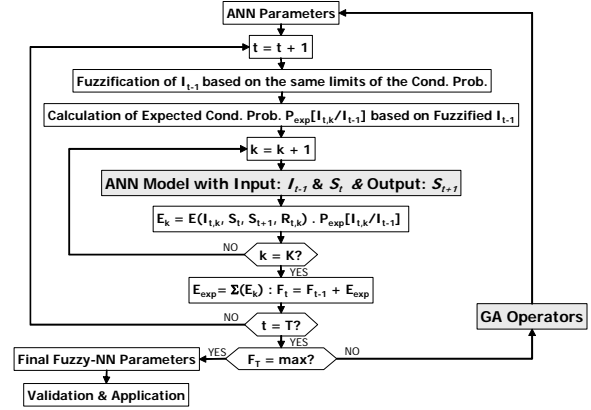


Fig 2. Flowchart of the calculation for SFNN

The SFNN can be applied to both types of conditional probabilities presented in the previous section. The values found through the traditional conditional probability, which uses the crisp intervals, can be directly used for the SFNN optimization model or they may be interpolated according to the actual previous inflow and the representative inflows. As for the CP of fuzzy events, the fuzzy representative inflows may be used. However, in doing so, the optimization model may result in optimum fuzzy values of release and end-of-period storage that are too difficult for users to understand and too vague for their practical consideration. The propagation of the fuzziness associated with these values, when applied to the optimization model, would tend to generate greater vagueness, resulting in less efficient operation rules. Therefore, a method that could be considered analogous to interpolation is proposed, in which only the center values are considered and according to the actual previous inflow, the conditional probability values is changed.

As the previous inflow value is known, based on the already calculated conditional probabilities of a fuzzy event, the new expected value can be obtained for each of the representative stochastic inflow  $I_{t,k}$ , which is also

defined as the center value of the fuzzy event  $k$ . The expected conditional probability for the SFNN model is calculated based on equation ( 10 ). The center values of the fuzzy membership function for the fuzzification of the previous inflow are the same as the ones used for the calculation of the conditional probabilities themselves, having the center and limit values defined as the (unconditioned) probability for each month as shown in the lower part of Fig 1.

$$P_{exp}(I_{t,k} / I_{t-1}) = \sum_{j=1}^J [P\mu(A_k / B_j) \cdot \mu(I_{t-1})] \quad (10)$$

Where  $P\mu$  is the conditional probability of a fuzzy event,  $\mu$  is the membership function or fuzzy grade value of previous inflow  $I_{t-1}$  and  $A$  and  $B$  two consecutive fuzzy events. An example of the calculation of the expected probabilities is shown below in Fig 3, where values in the center table represent the conditional probabilities, calculated as in the previous section, between two consecutive fuzzy events, i.e. previous and actual inflows. The stochastic fuzzy neural network (SFNN) is then trained considering the representative inflow values  $I_{t,k}$  and the respective expected conditional probabilities  $P_{exp}(I_{t,k}/I_{t-1})$ .

		Actual Inflow (t)							
		XL	VL	L	M	H	VH	XH	Sum
Previous Inflow (t-1)	XL	0.35	0.39	0.26	0.00	0.00	0.00	0.00	1.00
	VL	0.32	0.37	0.25	0.06	0.00	0.00	0.00	1.00
	L	0.00	0.22	0.32	0.38	0.07	0.01	0.00	1.00
	M	0.01	0.03	0.02	0.38	0.42	0.12	0.01	1.00
	H	0.03	0.15	0.25	0.17	0.21	0.12	0.06	1.00
	VH	0.00	0.11	0.26	0.16	0.21	0.16	0.10	1.00
XH	0.00	0.00	0.00	0.00	0.17	0.31	0.53	1.00	
Expected Probability		0.06	0.25	0.30	0.32	0.06	0.01	0.00	1.00

Fig 3. Example for the calculation of the expected probability  $P_{exp}$

Usually for the training of neuro-fuzzy models, the target vector is previously defined and usually accounts for observed values. However, in the case of optimization using the ANN based models; the training is proposed to be based directly on the maximization of the recursive function  $F$  itself. The following formulation are proposed for the recursive equations ( 11 ) and ( 12 ), considering two different types of fuzzy decision:

*Weighted average type*

$$F(S_t) = \max_{S_{t+1}} [E_{exp}(E_{t,k}) \cdot \alpha + F_{t-1}(S_{t-1})] \quad (11)$$

*Minimization type*

$$F(S_t) = \max_{S_{t+1}} [\min [E_{exp}(E_{t,k}), F_{t-1}(S_{t-1})]] \quad (12)$$

Where  $\alpha$  is the weight, here we consider all the weights to be the same and equal to  $1/T$ . Nevertheless, different weights may be incorporated to increase the importance of a particular period in time or as a discount factor, where  $T$  is the total number of stages (time-steps). The summation of  $\alpha$  is equal to 1, which guarantees the final value of the recursive function ( 11 ) to be between  $[0, 1]$ . This does not affect the final optimum storage or release sequence, but instead facilitates the comparison of the final results with other decision types, such as the minimization type ( 12 ).  $E_{exp}$  is the expected value of the evaluation function, as shown in equation ( 13 ), which depends on the expected probability.

$$E_{exp} = \sum_{k=1}^K E_k \cdot P_{exp}(I_{t,k} / I_{t-1}) \quad (13)$$

where the  $P_{exp}$  is the expected conditional probability presented in equation ( 10 ),  $k$  represents the index of conditional probabilities, referring to the representative stochastic inflows  $I_{t,k}$ .  $E_k$  is defined according to the kind of aggregation operator to be used, such as shown in equations ( 14 ), ( 15 ) and ( 16 ), for three aggregation operators: weighted-average, product and minimization, respectively.

$$E_k = \sum_{n=1}^N E_n \cdot \beta \quad \text{where} \quad \sum_{n=1}^N \beta = 1 \quad (14)$$

$$E_k = \left( \prod_{n=1}^N E_n \right)^{1/N} \quad (15)$$

$$E_k = \min_n (E_n) \quad (16)$$

Where  $\beta$  is the weight, here assume the same for all objectives but which could vary depending on the importance of each objective.  $N$  is the total number of fuzzy objectives and index  $n$  represents each fuzzy objective. All of them are represented by a fuzzy objective membership functions  $E_n$ . The fuzzy objective membership functions used here are shown later in Fig 7. The calculation of the expected value  $E_{exp}$  is used in the calculation of the recursive function shown in equation ( 11 ) and ( 12 ). Optimization results using the above fuzzy decisions and aggregation operators are shown

later.

Having a set of parameters for the neuro-fuzzy system, it is possible to obtain the end-of-period storage level  $S_{t+1}$ , as we know  $S_t$  and  $I_{t-1}$  values. For each stochastic inflow value  $I_{t,k}$  we can obtain a release value  $R_{t,k}$  based on the reservoir mass balance equation ( 17 ), which neglects other losses such as evaporation and infiltration that also could be considered.

$$R_{t,k} = S_t - S_{t-1} + I_{t,k} \quad (17)$$

This operation is carried out for the whole training period  $T$ . If the parameters of the neuro-fuzzy system are considered to be optimal, giving the maximum or quasi maximum value for the recursive function then the training process can be concluded. After validation the model can be finally applied as the network is now representing the reservoir stochastic optimal operation rules.

It is important to note that in the case of SDP models all alternatives are tested for a period of one year, or the time representing one cycle of the stochastic variable, here being the monthly inflow discharges. The results of a SDP model are presented as guidecurves that can be used to obtain the end-of-period storage when the actual month, previous inflow and initial storage values are provided. On the other hand, the SFNN model is training using directly observed data, considering the conditional probability between stages and also the long-term characteristics of river discharge. In the case that observations are scarce or considered to be incomplete, the SFNN model can be trained with artificially generated random or synthetic time series data. On the other hand, only pure random values can be used, however the long-term characteristics of inflow are no longer considered, which could decrease part of the SFNN model efficiency. It is important to keep in mind that the computational effort increases with an increase in the data set size. Nevertheless, for longer training data, more efficient operation rules may be expected. The optimal results found by SFNN are no longer presented as guidecurves, but instead as a fuzzy neural network model, stochastically trained, which has the end-of-period storage represented by its output neuron, and initial storage and previous inflow represented through its input neurons.

## 5. Fuzzy Neural Network Model

Artificial neural network (ANN) models have been widely used in various fields, such as aerospace, finance, robotics, environmental assessment and hydrology, for different purposes such as classification, pattern identification, simulation and prediction (Hagan *et al.*, 1995). The combination of ANNs with other techniques, such as fuzzy theory, has been proposed by some authors. The use of fuzzy theory with the ANN aims to combine the ability of fuzzy sets to represent knowledge that is understandable to human beings with the learning capability of ANNs (Jin, 2003). Therefore, a neuro-fuzzy system is a fuzzy system that can learn from data as well as from experts' experience. Application of fuzzy theory with ANNs may change the model from being a "black-box" into a "gray-box" type. As described by Jin (2003), there have been some attempts to develop neuro-fuzzy models and the most well known example is the adaptive-network-based fuzzy inference system (ANFIS) proposed by Jang and Sun (1993, 1995). The ANFIS model has been successfully applied to many problems, including storage reservoir operation and it can serve as a basis for the construction of fuzzy if-then rules. However, this method can face great computational effort, as the increase in the number of parameters is proportional to its number of linguistic variables and input nodes.

### 5.1 Network Architecture

Based on the above explanation, a simpler neuro-fuzzy network is proposed, whereas the basic architecture of the model is briefly presented as follows. The training process is carried out by a genetic algorithm (GA) model, where the objective function is the as the one used for the reservoir operation, such as the ones shown later in Fig 7. The developed neuro-fuzzy system architecture is shown in Fig 4.

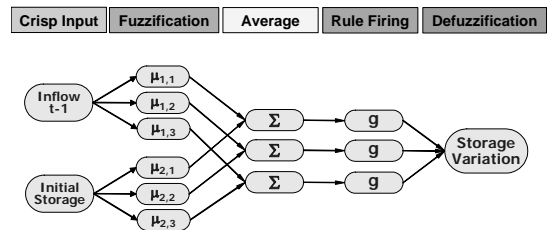


Fig 4. Architecture of the proposed neuro-fuzzy inference system

Previous inflow ( $t-1$ ) and initial storage are

represented by two input nodes, whereas the output node gives the variation of the end-of-period storage value, which is the consequence of the operating rules. Each month has its own network, which proved to significantly increase the optimization efficiency. Hence, the operation rules represented by the neuro-fuzzy system are analogous to the operation guide-curves derived by the SDP models, where by three types of information is required (actual month, previous inflow and initial storage).

## 5.2 Fuzzification of Inflows and Storage

Considering a crisp input value  $x_i$ , such as previous inflow and initial storage, the fuzzification function is carried out as shown in Fig 5. The parameters  $X_k$  are defined as the modal value of the fuzzy membership function of each linguistic variable  $k$ . For the case of previous inflow ( $t-1$ ), the parameters  $X_k$ , where  $k[1,2,\dots,7]$ , are defined based on the cumulative probability function for each month, accounting for the same values as the ones used for the calculation of the conditional probability of a fuzzy event, as shown in Fig 1. For the case of initial storage, the parameters  $X_k$  are optimized during the training process. The extreme values  $X1$  and  $X7$  are set equal to the minimum and maximum expected values, i.e. constrain values use for the reservoir operation here assumed equal to 600 hm<sup>3</sup> and 3250 hm<sup>3</sup>, respectively. Seven linguistic values are used as they proved to be enough for modeling the system. Moreover, this number has also been used widely in other applications of fuzzy inference. The resulted membership values are represented by the symbol  $\mu_{i,k}$ , which is represented graphically in Fig 5.

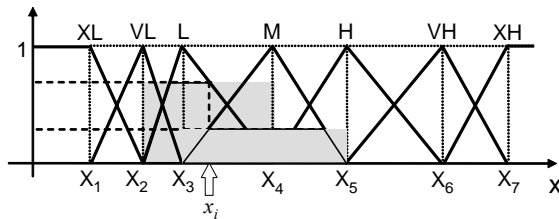


Fig 5. Membership function for the fuzzification of input variables  $x_i$

The resulted membership values  $\mu_{x,k}$  for each linguistic variable  $k$ , found after the fuzzification of the crisp input value  $x$ , are then combined. There are various methods used within fuzzy inference theory, such as product, minimization and maximization. However, after trying these different methods, the weighted sum is

found to be the most appropriate as robustness of the whole system increases. Probably, this is due to the use of triangular functions that present only two membership values other than zero, which in a product type aggregation could null all the membership values. The resulted fuzzy number  $\mu_{y,k}$  after the combination through the transfer function of previous units  $i$  is presented in equation ( 18 ).

$$\mu_{j,k} = \frac{\sum_{i=1}^N (W_{ij} \cdot \mu_{i,k})}{\sum_{i=1}^N W_{ij}} \quad (18)$$

Where  $N$  is the total number of previous units and  $W_{ij}$  is the weight between neurons  $x_i$  and  $y_j$ ,  $W$  is also optimized during training. The resulting fuzzy inference is represented by  $\mu_{j,k}$ .

Knowing the fuzzy value  $\mu_{i,k}$ , it is possible to obtain a crisp value  $y_j$ , which in our case represent the variation of final (end-of-period) storage, through many available defuzzification methods. The most common defuzzification methods are the center of gravity, center of sums and center of areas (Jin, 2003). A method based on the center of gravity and described in Jin (2003) is adopted here, in equation ( 19 ).

$$y_j = \frac{\sum_{k=1}^K (\mu_{j,k})^g \cdot Y_k}{\sum_{k=1}^K (\mu_{j,k})^g} \quad (19)$$

Where  $Y_k$  is the kernel or the modal value and is identified during training;  $g [0,\infty)$  is a power parameter responsible for “firing the rule”, which can be properly adjusted to obtain a more efficient performance for the defuzzification method. This parameter is also optimized during the training period. After calculating the variation of storage ( $dS = y_i$ ) and knowing the initial storage ( $S_{initial}$ ), the final storage ( $S_{final}$ ) can then be calculated by equation ( 19 ):

$$S_{final} = S_{initial} + dS \quad (20)$$

## 6. Case Study

Barra Bonita reservoir is located in the middle Tietê River basin, in the São Paulo State of Brazil, (22°29' S and 48°34' W) with maximum surface water at an altitude of 453 m. The location of the reservoir is shown

in Fig 6. The reservoir has a water surface area of approximately 340 km<sup>2</sup>, total volume of 3.6 km<sup>3</sup> and length of 50 km. Maximum and average depth are around 25 and 10 meters, respectively. Average water fluctuation of the reservoir is approximately 5 meters. Hydropower energy is the primary water use of the reservoir. Other uses include navigation, recreation, water supply and fishery production. The Reservoir is the first of a series of six reservoirs, around 300 km downstream from Brazil's biggest city, Sao Paulo. It can be classified as a subtropical/tropical reservoir with an intermediate retention time of around one to two months. Air temperature normally varies only by 15°C between winter and summer. The wet season occurs between September and March. Annual cumulative precipitation is around 1400 mm, with maximum wind velocity of 5 to 7 m/s during winter. The average annual flushing rate is 414 m<sup>3</sup>/s.

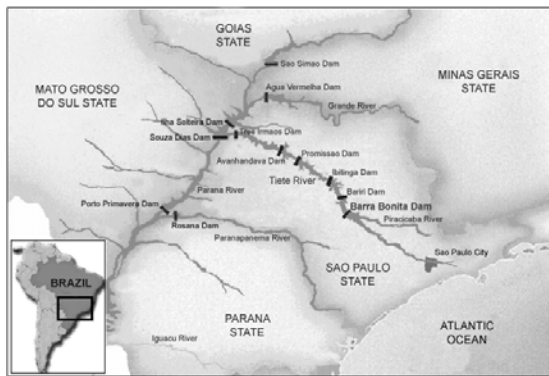


Fig 6. Location of Barra Bonita reservoir

### 6.1 Operation Objectives

The proposed methodology is applied in the development of a simple storage reservoir optimization model considering fuzzy objectives related to water quantity characteristics of the system. These functions are defined here as flow stabilization  $\mu_{stab}(R_t)$  or *STAB*, as a function of operated release, and hydropower generation (in KW)  $\mu_{pow}(R_t, S_t)$  or *POW*, as a function (indirectly) of operated release and storage level. Both objectives are represented by fuzzy membership functions, defined between 0 and 1, representing the low and high satisfaction of operation, respectively. The fuzzy membership functions represent the degree of satisfaction or conformity in attaining the operation objectives. Due to a lack of realistic information about the reservoir operation, the two objectives considered here for optimization have been hypothetically

formulated and their fuzzy membership functions are shown in Fig 7.

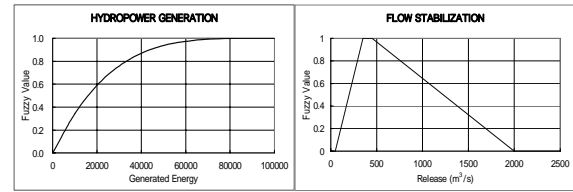


Fig 7. Fuzzy objective membership functions for the reservoir operation

The release values of 350 and 400 m<sup>3</sup>/s are assumed to represent maximum satisfaction in the flow stabilization membership function, which is intended to stabilize the release around the historical average inflow for the reservoir. It is very important, particularly in the case of Brazil, to consider power generation in the optimization. The membership function is based on the produced and demanded energy. Due to a lack of data regarding the energy production, an average for all months is used to represent energy demand. It may be improved with realistic data and generation targets for the specific reservoir. Another way to improve the power generation analysis is by using statistical analysis to predict demand. However, for the purpose of this study, a value of 100MW is assumed as the monthly average demand for all months. The generated energy (KW) is calculated based on the release  $R$  (m<sup>3</sup>/s) and water head  $H$  (m) by equation ( 21 ), where  $H$  is a function of the average of initial- and end-of-period storage values of stage  $t$ .

$$Generated\ Energy = 8.319 \cdot R_t \cdot H_t \quad (21)$$

### 7. Validation of the SFNN Model

To test the proposed methodology, the Barra Bonita reservoir is optimized through five different optimization schemes, including the Stochastic and deterministic FNN method and three SDP formulations, all for monthly time-steps. The total time series is divided into two data sets, where 22 years of observations are used to identify the conditional probabilities for all models and to train the SFNN model, whereas 11 years is used to validate the proposed methodology in a practical reservoir optimization. All the optimization schemes use the same fuzzy objective membership functions presented in Fig 7, which are combined using the product aggregation



method and the weighted-average type of decision, for the same length of optimization horizon. The first scheme is a simple deterministic DP model where the inflow in stage  $t$  is considered to be known. Therefore, the DDP is expected to give the maximum satisfaction for the operation objectives. The recursive equation for the DDP is presented in equation ( 22 ).

$$F_t(S_t) = \max_{S_{t+1}} \{E_t(\cdot) + F_{t+1}(S_{t+1})\} \quad (22)$$

Where  $F$  is the recursive equation,  $E$  is the evaluation function and  $S$ ,  $R$  and  $I$  represent storage, release and inflow, respectively, for time step  $t$ .

To exemplify the importance of storage discretization within the DP optimization, some levels of storage discretization were tested for the optimization of the Barra Bonita reservoir applying the deterministic formulation, considering the whole inflow data set (33 years). The final results of the recursive equation for the whole optimization horizon (referred to as average in the legend of Fig 8), the minimum values among all  $E_t$  values and its standard deviation are presented in Fig 8.

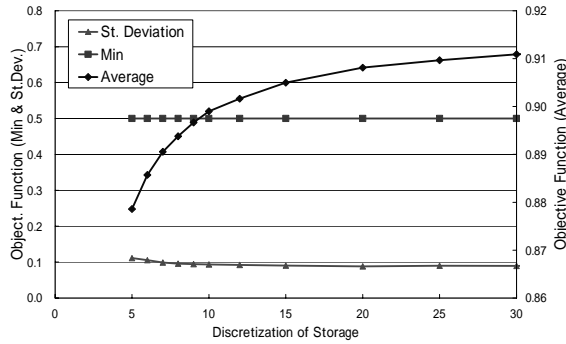


Fig 8. Relation between storage discretization and DP optimization results

Then, stochastic characteristics of inflow are considered by applying two SDP models. The first one is based on discrete conditional probability and the second uses the proposed fuzzy formulation. This is to confirm the efficiency of the proposed conditional probability of a fuzzy event. The recursive equations for the two stochastic optimization schemes using SDP are presented in equations ( 23 ) and ( 24 ), respectively.

$$F_t(S_t, I_{t-1}) = \max_{S_{t+1}} \left\{ \sum_{k=1}^K [E_t(\cdot) + F_{t+1}(S_{t+1}, I_t)] \cdot P(I_{t,k} / I_{t-1}) \right\} \quad (23)$$

Where  $P$  is the conditional probability for the discrete inflow intervals  $k$ ,  $I_{t,k}$  is the stochastic inflow which refer

to the cumulative probability values and  $E_t(\cdot)$  is the evaluation function.

$$F_t(S_t, I_{t-1}) = \max_{S_{t+1}} \left\{ \sum_{k=1}^K [E_t(\cdot) + F_{t+1}(S_{t+1}, I_t)] \cdot P\mu(I_{t,k} / I_{t-1}) \right\} \quad (24)$$

Where  $P\mu$  is the fuzzy conditional probability which has been already defined in equation ( 9 ). The state transition function has been presented in equation ( 17 ). Note that this is the inverted form of the mass balance equation (Labadie, 1993). This form is chosen for its coding simplicity. The optimization is subjected to the constraint of minimum and maximum acceptable values of storage and release. Moreover,  $F_T = 0$ , where  $T$  is the last chronological time, and therefore, for the backward calculation, the first stage used for calculation.

Since the transition probabilities repeat every 12 months, the undiscounted stochastic DP calculations are repeated via successive approximations to confirm and guarantee that the optimum guidecurves of end-of-period storage for each month are converging to stationary values. As a result, optimal guidecurves may be applied to each year over the entire operational horizon for any sequence of inflow. As referred by Labadie (1993), if this procedure converges, then the solution must be optimum. Finally, the reservoir is optimized through the SFNN proposed method, also considering the conditional probability of a fuzzy event. The SFNN recursive equation has already been shown in equation ( 11 ). Note that both recursive equations for the SDP models are presented as backward-moving while the SFNN can be formulated for the forward-moving.

## 8. Results

Fig 9 shows the results of cumulative conditional probability curves for the month of February carried out for the monthly inflow discharge of the Barra Bonita reservoir using the CP for discrete intervals of inflow, whereas Fig 10 shows the results after using the conditional probability of fuzzy inflow.

It can be seen that the results of the fuzzy based method (Fig 10) are much smoother than the ones from the formulation based on discrete intervals. Therefore, they are much more in keeping with what is expected of realistic conditional probability curves. Moreover, results after optimization using a SDP model showed to be more efficient than the ones using the traditional method, as can be seen in Fig 11.

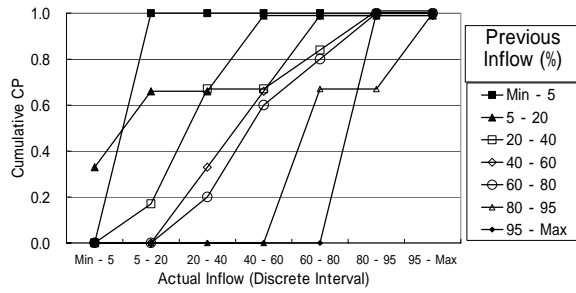


Fig 9. Cumulative CP assuming discrete intervals

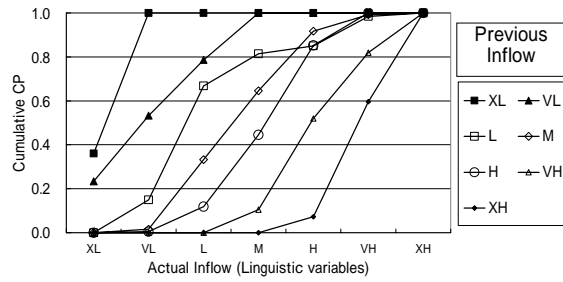


Fig 10. Cumulative CP assuming fuzzy linguistic variables

Fig 11 shows a comparison of evaluation function results after the five different optimization schemes using the weighted-average aggregation operator for the objective functions, equation ( 14 ): a deterministic DP (DDP) model (maximum expected value, for which inflows are considered to be known), a deterministic FNN model (maximum expected value, for which inflows are also considered to be known), SDP model using both method (discrete and fuzzy interval) for the calculation of conditional probabilities and the proposed SFNN model (considering the CP of a fuzzy event).

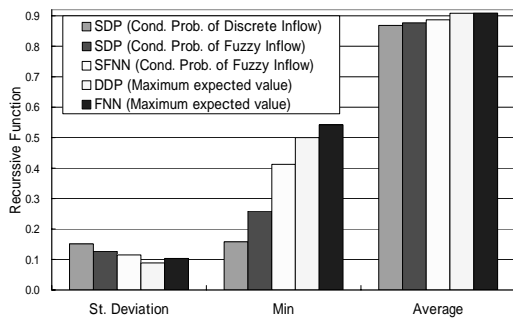


Fig 11. Results after five optimization schemes (weighted-average aggregation)

It can be seen that the SFNN model presents a mean value slightly superior to the ones found with the SDP models, showing the increase in the efficiency of optimization when the SFNN model is compared to the other two SDP formulations (as values near to 1 mean higher satisfaction and consequently more efficient operation rules). Moreover, the minimum value of the evaluation function of the SFNN is greater, and its standard deviation is lower than the SDP models, also confirming the efficiency of the proposed method. Within the two SDP models, it is possible to conclude that the use of CP of fuzzy inflow improved the operation performance, as the “average” value was higher (closer to 1) when using the SDP based on the CP pf fuzzy event. The DDP and FNN formulations are only intended to show what would be the maximum expected results that could be obtained if future inflows were known exactly. The determinist optimization using the same FNN architecture also showed excellent results compared to its counterpart DDP.

The final storage and release sequences after the SFNN optimization are shown in. It can be seen that the proposed SFNN operation model is flexible enough due to discretization of storage levels based on fuzzy inference, which can calculate any storage level independent of limitations originating from the discretization process, usually observed in SDP models. Also in Fig 12, the training and validation periods used for all schemes are shown, which correspond to 264 and 132 months, respectively.

Fig 13 shows the behavior of the objective function throughout the generations of the GA optimization model. As expected, there is a great increase within the first generations and slower improvements in the optimal values towards the last ones.

Besides the previous formulation, another one was carried out for the same optimization schemes as for the previous case, but now the product aggregation operator, equation ( 15 ), was considered. This second test was necessary to guarantee that the proposed model is robust enough, independently of the operator being used. In this case as well, the SFNN model showed to be superior to the SDP schemes as can be seen in Fig 14.

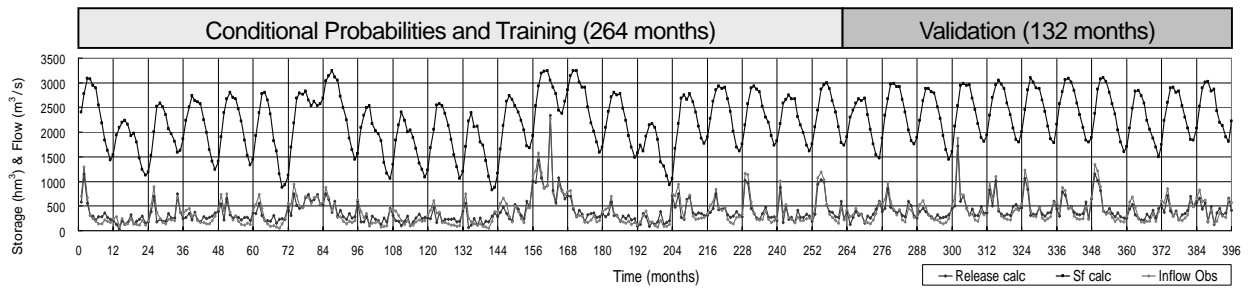


Fig 12. Results of operation using the SFNN model

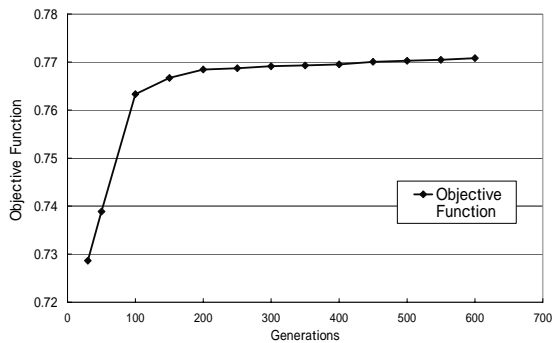


Fig 13. Results of objective function versus GA generations

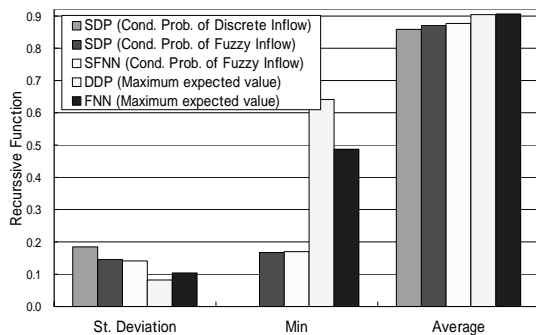


Fig 14. Results from five different optimization schemes (product aggregation)

Within the two deterministic schemes, where inflows are assumed to be known even for the future (validation period), the FNN formulation also showed more efficient results for the final recursive function (“average” in Fig 14). The standard deviation (“st. deviation”) and minimum value (“min”) are calculated based on the whole validation horizon (132 months) as done before in Fig 11.

Finally, we investigate how the proposed model responds to different recursive functions. The optimization is carried out using six different SFNN formulations depending on the combination of two decision types (minimization and weighted-average) and

three aggregation operators (minimization, weighted-average and product). The operation results for the training and validation periods are condensed as cumulative probability function, as shown in Fig 15 and Fig 16, to create a better visualization of the final results after finding the stochastically trained optimized operation rules.

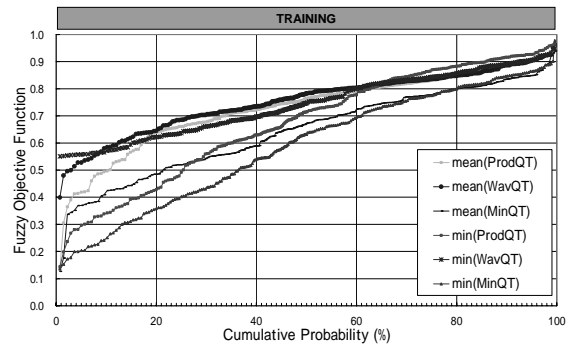


Fig 15. Objective function results after six SFNN formulations – training

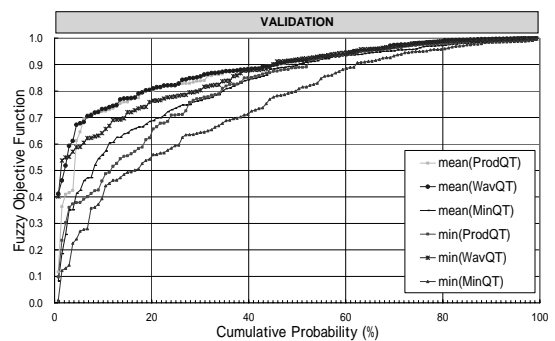


Fig 16. Objective function results after six SFNN formulations - validation

It can be seen that when using the minimization operator, the overall results were moderately decreased. For our case study, the stochastic training of the neuro-fuzzy system using such operators seems to be less robust, because, even though the minimum value could be maximized for the training period, in the validation

period the operation policy represented by the SFNN was not able to return the maximum of the minimum among the other results. That is to say, for the validation period, the resulted minimum value was not the minimum when using the minimization operator, which was not really expected, as this operator was supposed to maximize the minimum values. This problem may also occur with other stochastic optimization techniques as well, such as SDP. Fig 17 shows a comparison of the minimum results found after training and validation for the six SFNN formulations. When looking at Fig 17, the results should be compared within the same aggregation operator for the two periods, training and validation.

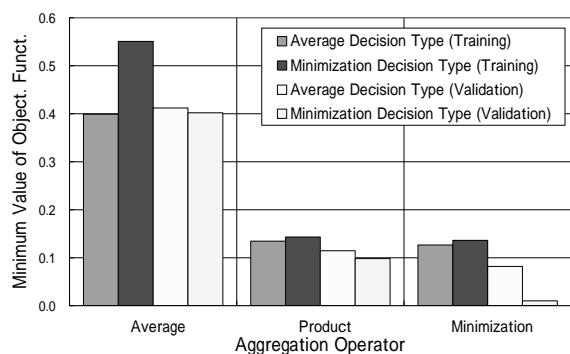


Fig 17. Minimum value of objective function for six SFNN formulations

Basically, it may be said that the decision on which aggregation method and fuzzy type of decision to use depends significantly on the judgment of operators. Even though in our problem the minimization decision type did not show great robustness, in other problems with other operation purposes and different shapes for the fuzzy objective membership functions, the minimization can also find its use. This decision type demonstrated an inability to reflect any correlation among intermediate evaluation values within the optimization (training) period, as it always chose the smallest among different evaluation values, which in some operation problems may be insufficient for properly achieving efficient operation rules. This may be aggravated when a greater number of objectives and longer training periods are used. The reason is simple, a water resources system such as reservoir operation like many other systems, is designed to present the least risk of failure as possible. However, systems presenting risk “zero” of failure may be physically and economically impossible to achieve. Therefore, if one objective in a certain time of the operation horizon, cannot be satisfied at all (membership

function value equals zero) the whole system is assumed to fail. That is why the minimization type is frequently called the pessimistic decision. It is recommended that when using the SFNN for deriving operation rules for practical use, various types of objective functions, aggregation operators and decision types should be tested. Nevertheless, it could be said that as harder objectives are difficultly satisfied during the training period, then the calculated operation rules would be less efficient.

## 9. Conclusions

First, the conditional probability of a fuzzy event proved to increase the efficiency of the stochastic DP. This was possible as a result of the use of more realistic conditional probability curves, found through the fuzzy methodology presented here. This was verified after a comparison of the results from the two SDP optimization schemes. The CP considering stochastic representative inflows as fuzzy events is believed to reduce some of the uncertainties related to the incomplete data set and the crisp discretization process when calculating the conditional probability of discharge values.

The results from the proposed SFNN model showed improvements in optimization efficiency, which could be obtained as a result of the introduction of fuzzy inference to deal with storage discretization. Moreover, the SFNN model has the ability to consider, not only the conditional probability but also, the long-term characteristic of inflow, as the model is trained with actual inflow observation data. This is different from the SDP model which tries all combinations related to the discretized storage levels for a twelve-month period only, ignoring the long-term characteristics of inflow. Moreover, the SFNN also addresses the uncertainties of the conditional probability as the model is prepared to calculate a new expected conditional probability for each time stage  $t$ , through a kind of interpolation (based on the membership functions) of the CP curves.

Different from the SDP, the forward-moving calculation scheme for the SFNN model was successfully carried out, making the combination of the latter with other models straightforward, such as runoff and water quality models. The possible combination with other models may increase the efficiency of optimization models and may expand its applications for optimization of more complex systems, which for instance may involve more state variables, such as the one presented in

the next chapter.

Even though further research is necessary, having the whole simulation being possible at once after the parameters of the neuro-fuzzy system are defined, the application of SFNN seems to present great potential to overcome the problem related to the curse of dimensionality, commonly faced by SDP models, in the case of many state and decision variables. For example, in the case of multi-reservoir systems, new state variables could simply be added as new neuron units.

The results, using different fuzzy decision and aggregation operators, indicated that the SFNN is flexible enough to optimize different types of recursive functions, demonstrating great potential for its application to a variety of other kinds of stochastic optimization problems. As could happen to other techniques as well, the minimization decision type may present less robust results when optimization is carried out for systems with a greater number of operation goals and longer optimization horizon. Moreover, the shape of the fuzzy objective function may also influence the performance of the model when using the minimization and product decision types.

After proving the efficiency of the proposed model for the stochastic optimization of the proposed problem, considering only quantity objectives for the reservoir operation, in the next chapter, a more complex problem, considering water quality objectives is considered.

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## ニューロ・ファジーシステムを用いた多目的貯水池の確率論的操作

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### 要旨

本研究では、確率的ニューラルネットワーク (SFNN) と名付けたシステム最適化方法を新たに提案する。それは、推計学的に習練されたニューロ・ファジーシステムであり、準最適解を生み出せる方法として定義されている。この方法は、必要性のある後退スキームをはじめとしてSDPモデルに関する諸問題を克服するために考案している。本提案方法の検証を目的として、確率変数を流入量として、実ケースの貯水池最適化と操作を用いて考察した。同一の最適化問題について本提案手法と他のDPアプローチとで結果を比較したところ、SFNNは最適化の結果に改善が見られた。さらに、確率的流入量を見出す際には、流入量の離散化における不確実性を扱うためにファジー事象の条件付確率の利用を提案した。この貯水池操作の多目的最適化においては、他の目的と容易に比較できるようにファジー論理を扱った。

キーワード: 確率的最適化, ファジーニューラルネットワーク, ファジー事象の条件付確率, 貯水池操作

