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Rainfall Prediction Considering Error Structure of Predicted Rainfall

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Synopsis

Rainfall field is predicted using radar observation translation vector model, and prediction error structure is analyzed with the predicted and observed rainfall fields. With the analyzed error characteristics, random error fields are simulated by covariance matrix decomposition method. The simulated random error fields successfully contain the error structure. The simulated error fields offer prediction reliability in the form of error range which is formulated by standard deviation of the generated error values on each grid.

Keywords: Radar rainfall prediction, Error structure, Random error field

1. Introduction

Currently, there are many hydrological models which can properly simulate basin characteristic if parameters of the model are optimized to a subject basin. In the case, accuracy of the simulation results which usually represent as a discharge hydrograph is mostly controlled by accuracy of rainfall data. This important roll of input rainfall data becomes more severe when the simulation is on real-time basis and so the input data is predicted rainfall data. But, it is still hard to say that the accuracy of the predicted rainfall is high enough to give confidence to the simulation results.

When we take into consideration of uncertainty in predicted rainfall data, it would be useful to provide accuracy of the predicted rainfall. During short-term rainfall prediction with any prediction model, if an error structure of predicted rainfall is properly analyzed with past prediction results and effective error factors are found, the error structure would be very useful to the present rainfall prediction. The analyzed error structure can be used for improving prediction methods or for figuring out reliability of the predictions. Whatever the error structure is for, important point to consider is that the prediction rainfall error has temporal variance and also spatial variance.

In this research, radar rainfall is predicted using translation vector model, and error structure of the predicted rainfall field is analyzed to get its statistics and spatial correlation coefficient. And then prediction error fields are simulated as a spatially correlated random error field according to the analyzed error structure. The final purpose of this research is to produce extended prediction rainfall field which offer prediction with its reliability in the form of probable error range.

2. Prediction and Prediction Error

2.1 Translation Model

Translation model (Shiiba *et al*., 1984) is used for short-term radar rainfall prediction. In the translation model, the horizontal rainfall distribution, $z(x,y,t)$ with the spatial coordinate (x,y) at time t is modeled as:

$$\frac{\partial z}{\partial t} + u\frac{\partial z}{\partial x} + v\frac{\partial z}{\partial y} = w \tag{1}$$

here, $u = \frac{dx}{dt}, \quad v = \frac{dy}{dt}, \quad w = \frac{dz}{dt}$

where, u and v are advection velocity along x and y, and w is rainfall growth-decay rate. The translation model assumes u, v, and w are having the forms:

$$\begin{aligned} u &= c_1 x + c_2 y + c_3 \\ v &= c_4 x + c_5 y + c_6 \\ w &= c_7 x + c_8 y + c_9 \end{aligned} \tag{2}$$

The values of parameters c_1, c_2, …, c_9 are identified by the least square method using observed radar rainfall data and they are updated on real-time basis. In this research, three observed rainfall fields, which have 3 km and 5 minutes of observe resolution, are used to determine u and v. When forecasting rainfall fields, the u and v are assumed to be spatially constant, but vary temporally. The growth decay rate w is always assumed to be zero.

Radar rainfall event used here is observed at Miyama radar station of Kinki area on 1990/9/12 and 9/13. Observation is available with 5 minutes interval, and prediction is done at every 5 minutes for one hour ahead. Average rainfall intensity of observed one and prediction one are shown in Figure 1.

2.2 Error Structure Analysis

After 1hr prediction is done by the translation model, absolute prediction error E_{ai} on grid i is calculated from difference between predicted rainfalls R_{pi} and observed rainfalls R_{oi} on the grid i.

$$E_{ai} = R_{oi} - R_{pi} \tag{3}$$

Figure 2 shows statistics of the prediction error. Average of the error is distributed around zero while standard deviation varies as time pass. The variation of standard deviation has a relation with rainfall intensity when it is compared with Figure 1. If there is a correlation between the standard deviation of the error and the prediction rainfall intensity, the prediction reliability can be estimated in the form of standard deviation of the prediction error. Frequency distributions of E_a show a normal distribution pattern (Figure 3). Here, the statistics and the distributions are calculated only with non-zero prediction error.

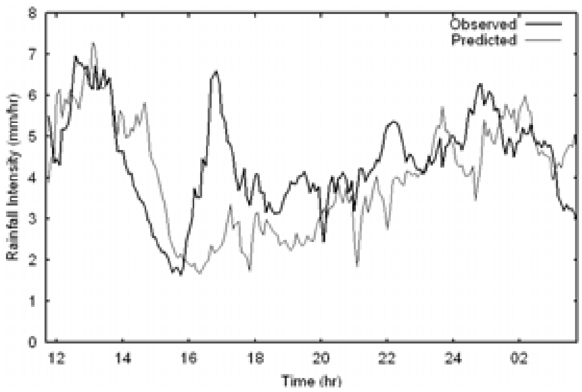

Fig. 1 Observed and predicted rainfall intensities

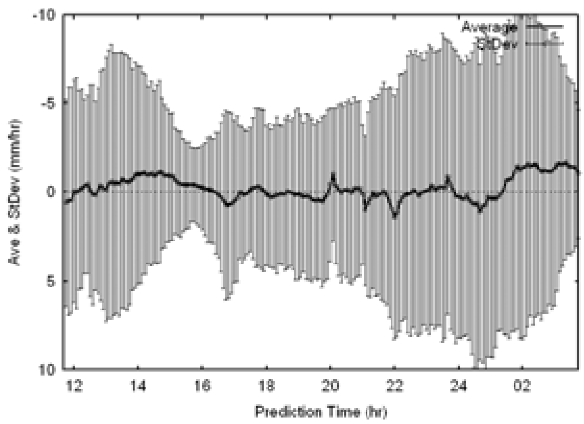

Fig. 2 Statistics of prediction error (1hr prediction)

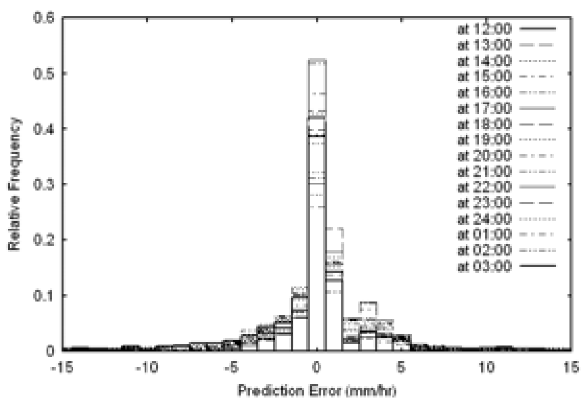

Fig. 3 Frequency distribution of the *Ea*

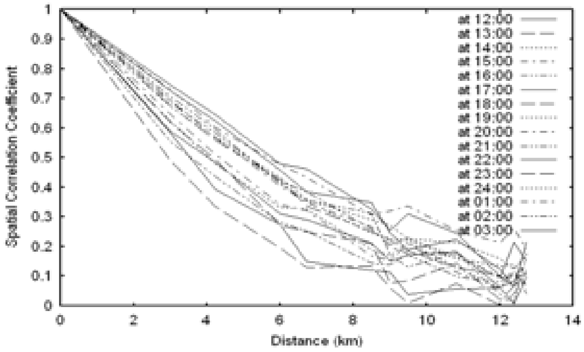

Fig. 4 Spatial correlation coeff variance along distance

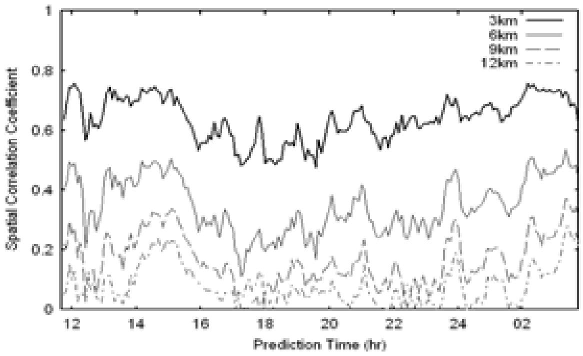

Fig. 5 Spatial correlation coefficient variance along time

Figure 4 and Figure 5 show spatial correlation coefficients from the prediction error fields. It shows how much the prediction error is spatially correlated to each other. Each coefficient is calculated by grouping each pairs of error which apart one grid for 3km, two grids for 6km, etc. The coefficient variance along distance (Figure 4) shows that effective range of the correlation is within 10 km. The coefficient variance along time (Figure 5) shows it also has a relation with rainfall intensity. The spatial correlation coefficients also can be estimated by using predicted rainfall intensity if we need to consider its time variance.

3. Producing Prediction Reliability Range

3.1 Random Error Fields Simulation

Matrix decomposition method is used to simulate random error fields which contails the analyzed error structure. Let's assume predicted error field vector Y is composed of a matrix B, which has a spatial correlation characteristic, and a random value vector x; $Y=Bx$. Here, the random values x are uncorrelated each other but follows a given probability distribution. Then a covariance matrix of the vector Y would be same as a square of the matrix B. (Tachikawa *et al.*, 2003)

$$\boldsymbol{E}[YY^{\mathrm{T}}] = \boldsymbol{E}[Bxx^{\mathrm{T}}B^{\mathrm{T}}] = B\boldsymbol{E}[xx^{\mathrm{T}}]B^{\mathrm{T}} = BB^{\mathrm{T}} \qquad (4)$$

After getting the matrix B from the BB^{T} using matrix decomposition method, the error field vector Y can be simulated with the matrix B and the random value vector x.

Figure 6 shows example of simulated random error field simulated by the covariance matrix decomposition method. By simulating many random error fields, probable error range of the prediction on each grid can be generated as shown in Figure 7. Statistics and spatial correlation coefficients for the simulated random error field is using the values of 14:00 on 13th Sep. Averages and standard deviations of the Figure 7 come from one hundred simulated random error fields when "-0.5993" for average and "7.7240" for standard deviation are given to the random error field simulation.

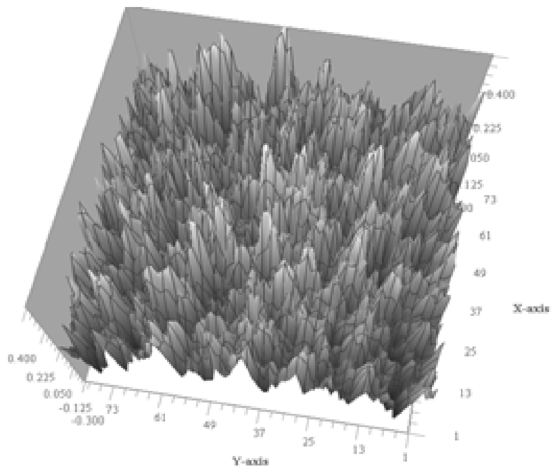

Fig. 6 Example of simulated random error field

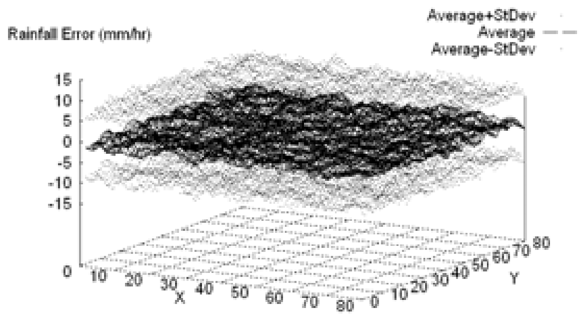

Fig. 7 Average and standard deviation on each grid calculated from 100 simulated random error field

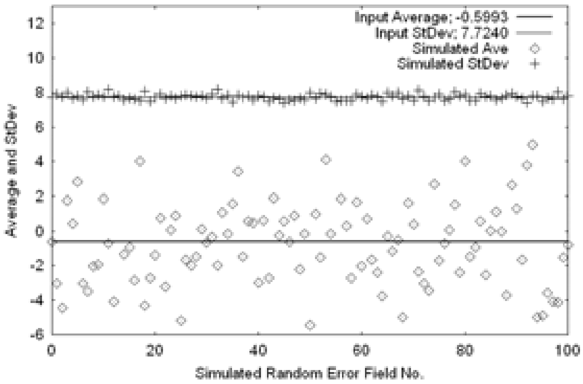

Fig. 8 Input and simulated values of Average and StDev

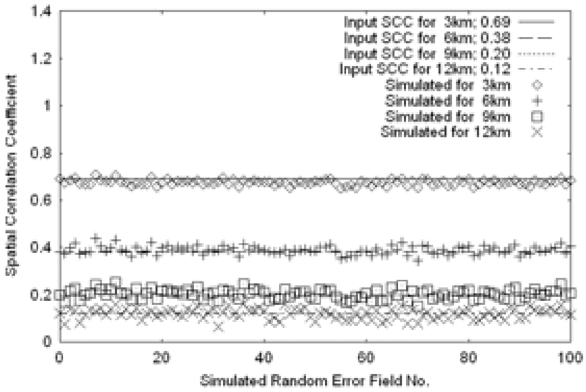

Fig. 9 Input and simulated values of spatial corr. coeff.

As shown in the Figure 8, each random error field is successfully reflecting input statistic values. Also Figure 9 shows the generated one hundred random error fields successfully reflecting input spatial correlation coefficient data.

3.2 Prediction Reliability Range

Deterministic prediction rainfall field from the translation model would be extended to probable prediction rainfall fields by joining with the simulated random error fields. The error range generated by many error fields would offer prediction reliability range in the form of standard deviation of the prediction errors. Successfully generated prediction error range is supposed to include observed rainfall field in the range.

Several problems occur during the extended prediction field generation procedure. Figure 10 shows schematic drawing of one rainfall field section of prediction with its error range and observed rainfall intensity.

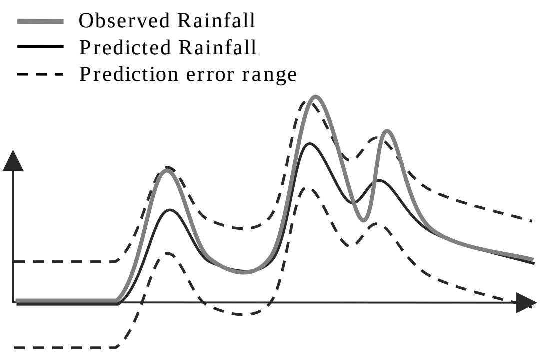

Fig. 10 Observed and prediction rainfall with its error range

Main problem is variant amount of prediction error according to different rainfall intensity. While the simulated random error fields make uniform error range over the prediction rainfall field, high amount of prediction error is found at high rainfall intensity area and low amount or no prediction error is found at low rainfall intensity area. Furthermore, the uniform prediction error range would be applied to no rainfall area and it makes unnecessary error range. Another problem which is also related to uniform error range is minus rainfall values of extended prediction rainfall fields. Even though minus values are unavoidable when we consider the possible error range, minus rainfall is not acceptable physically and it may cause malfunction if this extended prediction rainfall fields are applied to hydrologic model for runoff simulation.

4. Conclusion and Further Research

In this research, radar rainfall field is predicted using the translation vector model, and prediction error structure is analyzed spatially and temporally with the predicted and observed rainfall fields. The analyzed error structure includes average and standard deviation of the prediction error, probability distribution pattern, and spatial correlation of the error. Then random error fields are simulated by a covariance matrix decomposition method. The simulated random error fields successfully reflect the analyzed error structure.

The simulated random error fields offer prediction reliability in the form of error range which is formulated by standard deviation of the error values on each grid. But, the error range producing method introduces several problems to be considered. These problems are;

a. Different error range according to rainfall intensity
b. Uniform error range applied to no rainfall area
c. Minus rainfall values caused by error range

There are possible solutions to the uniform error range problem. First one is using accumulated rainfall field information. As time pass and as prediction error is accumulated, the accumulated error field shows a pattern rather than randomly distributed. There is certain area which has plus or minus value by the accumulated prediction error as shown in Figure 13 which means those area has been plus or minus prediction error continuously. Correlation coefficient of accumulated error field (Figure 13) and each error field (Figure 14) has rather low value as shown in Figure 11, the accumulated error pattern would be useful to give flexibility to the prediction error range.

Another possible solution to the uniform error range problem is to calculate each statistics on each grid using prediction error time series of its own grid. In that case the variant error range according to rainfall intensity would be automatically simulated. Also unnecessary error range on no-rainfall area would not happen if that area does not have predicted rainfall and prediction error.

The extended prediction by random error field introduced here can offer prediction reliability in the form of probable error range.

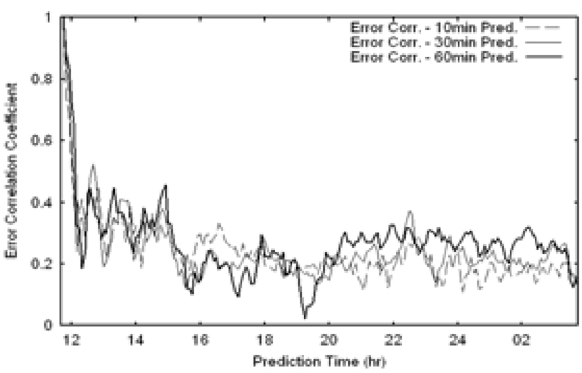

Fig. 11 Correlation coefficient of prediction error and accumulated prediction error on each time step

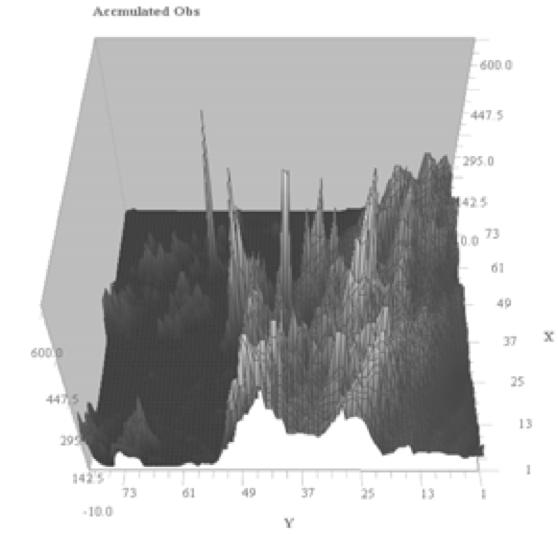

Fig. 12 Accumulated observed rainfall at 24:00 on 12^{th} Sep.

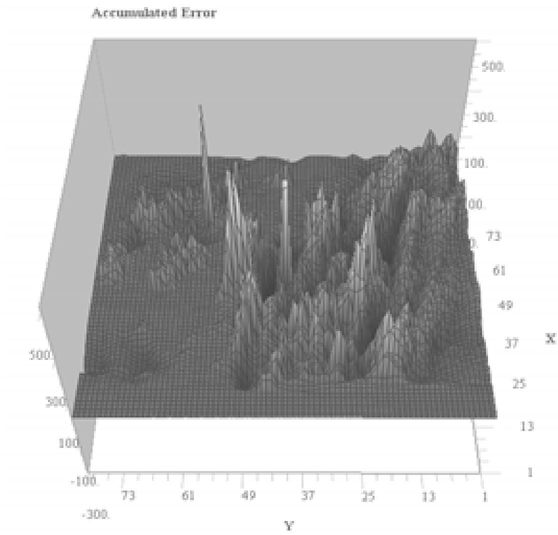

Fig. 13 Accumulated prediction error at 24:00 on 12^{th} Sep.

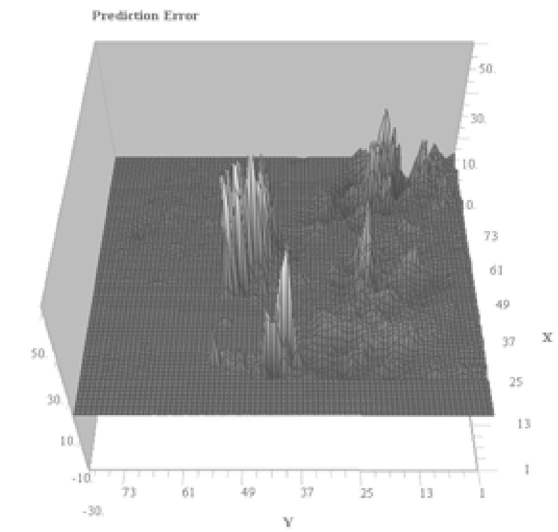

Fig. 14 Prediction error at 24:00 on 12^{th} Sep.

References

Shiiba, M., Takasao, T. & Nakakita, E. (1984) Investigation of short-term rainfall prediction method by a translation model, *Proc. 28th Annual Conf. on Hydraulics, JSCE*, 423-428.

Tachikawa, Y., Komatsu, Y., Takara, K. & Shiiba, M. (2003) Stochastic modeling of the error structure of real-time predicted rainfall and rainfall field generation, in *Weather Radar Information and Distributed Hydrological Modelling* (ed. by Y. Tachikawa, B. E. Vieux, K. P. Georgakakos & E. Nakakita), IAHS Publ., no. 282, pp. 66-73.

予測の誤差構造を考慮した降雨予測

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要　旨

移流モデルとレーダー観測値を用いて降雨を予測し，予測した降雨場の予測誤差構造を分析した。次に，予測誤差の空間場を確率場として捉え，分析した誤差特性に従って，共分散行列を分解する手法により誤差の空間場を生成した。模擬発生した誤差の空間場は，実際の予測誤差構造を適切に反映している。この手法は，グリッドごとに誤差分散として予測の信頼度を提供している。

キーワード: 実時間流出予測，状態更新，分布型流出モデル