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Real-time Updating of State Variables in a Distributed Hydrological Model

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Synopsis

When a distributed hydrological model is used for real-time flood prediction, how to update the spatially distributed state variables is an important issue to obtain accurate prediction. This study introduces three methods to update the state variables in a distributed hydrological model during a simulation. The introduced methods here, two kinds of ratio method, consider a spatial distribution of hydrologic variables in a basin. With the ratio method, state variables are updated successfully and better simulation results can be obtained than when the variables are not updated.

Keywords: real-time runoff prediction, updating state variables, distributed hydrological model

1. Introduction

To get a good rainfall-runoff simulation result, it is important not only to set model parameters precisely but also to give proper initial state variables. Furthermore, if there is a big difference between simulated discharges and observed one in a real-time simulation, updating parameters or state variables using observed values may help to improve forecasting accuracy. State variables in most of the distributed hydrological models are defined as storage amount of basin and its spatial distribution.

Updating state variables during a simulation is a difficult job, especially when using distributed model, because considerable amount of storage is already distributed all over the basin and it is almost impossible to measure that. Thus, when we use a distributed hydrological model for real- time flood prediction, how to update the spatially distributed state variables is a quite important issue to obtain accurate prediction. Also, the variables updating is an essential pre-step for data assimilation or filtering in a real-time simulation.

This study introduces several methods – steady-state method and ratio methods – to update spatially distributed storage amount during simulation in a distributed hydrological model. Each method performs separately on specific time steps in two flood events. Through different updating conditions with variable event types and time steps, compatibility and efficiency of the methods are checked. Finally, all methods are compared.

The grid-cell based distributed runoff model is used for this study, which is developed at Flood Disaster Research Laboratory, DPRI, here Kyoto University. The state variables are water depth at all computational nodes. Water depth is converted to storage amount by multiplying a basin area.

Program downloading is available at http://fmd.dpri.Kyoto-u.ac.jp/~flood/cellModel/cellModel.html.

2. Brief Description of the Hydrologic Model

The grid-cell based distributed runoff model solves the Kinematic wave equation using Lax Wendroff scheme on every node in a cell (Kojima et al., 2003). Discharge and water depth propagate to the next cell according to predefined routine order determined in accordance with DEM data.

The advantage of the grid-cell based model is that stage-discharge relationship of each cell reflects the topographic and physical character of cell. Specified stage-discharge relationship, which incorporates saturated-unsaturated flow mechanism, is included in each cell (Tachikawa et al., 2004). Because of variant slope and roughness coefficient, each cell has its own relationship.

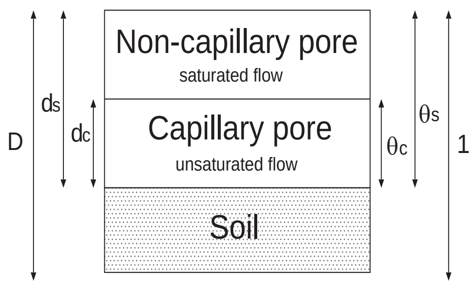

Fig. 1 Schematic drawing of soil layers supposed in the model.

The stage-discharge relationship is expressed by three equations corresponding to the three soil layers as shown in Fig. 1. When the water depth, h, is lower than the depth of unsaturated part ($0 \le h < d_C$), flow is described by Darcy's law with a degree of saturation, $(h/d_c)^{\beta}$, and a proportionality constant, k_c. If the h increases, flow from saturated part is considered with a different proportionality constant, k_a. After the water depth is higher than soil layer, d_s, overland flow is added by using the Manning's equation. According to this mechanism, the equations between discharge per unit width, q, and water depth, h, are formulated as Eq. (1). Also, Fig. 2 shows the graphical relationship between q and h. More details on the specified state-discharge relationship can be found in Tachikawa et al. (2004).

$$q(h) = \begin{cases} v_C d_C \left(\dfrac{h}{d_C}\right)^{\beta}, (0 \le h < d_C) \\ v_C d_C + v_a (h - d_C), (d_C \le h < d_S) \\ v_C d_C + v_a (h - d_C) + \alpha (h - d_S)^m, \\ (d_S \le h) \end{cases} \quad (1)$$

where, $v_C = k_C i$, $v_a = k_a i$, $\alpha = \sqrt{i/n}$.
n : roughness coefficient
i : slope of grid.

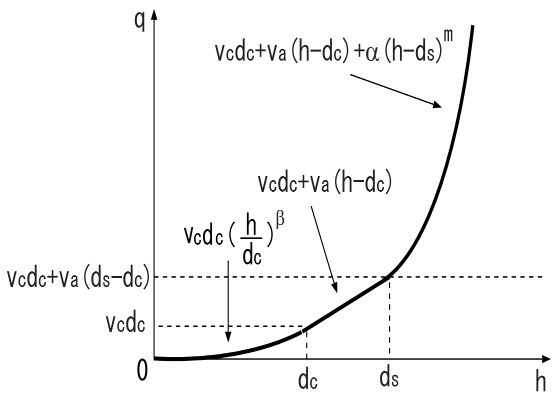

Fig. 2 Relationship between q and h.

The model is applied to the Kamishiiba Dam basin (area: $211.0km^2$) of Kyushu area for two flood events; Event 1 (97/9/15~9/19) and Event 2 (99/6/24 ~7/3).

3. State Variables Updating

3.1 Steady-state method

The simplest method to update the state variables by observed discharge is stated in Eq. (2).

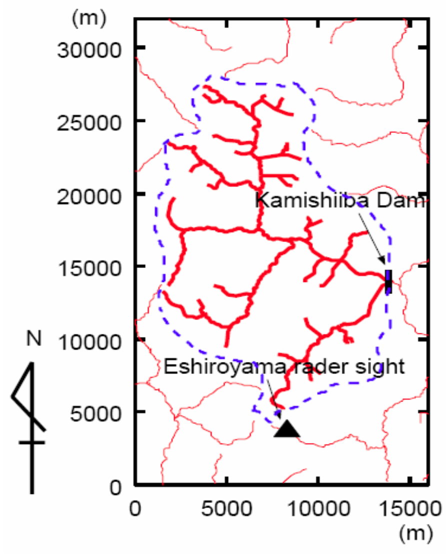

Fig. 3 Kamishiiba basin. Solid and dashed lines represent channel network and catchment boundary respectively.

When the grid-cell based distributed runoff model starts simulation, discharge from any cell is estimated by assuming steady-state condition. The discharge from any cell is set proportionally to the number of upstream cells as defined by the Eq. (2). The discharge is then converted to water depth using the stage-discharge relationship of each cell. This method is referred to as Steady-state method.

$$Q_i = QI \times \frac{UCN_i}{TCN} \qquad (2)$$

where, Q_i : discharge from any cell

QI : discharge at the outlet of basin

UCN_i : number of upstream cell at i^{th} cell

TCN : total cell number in the watershed.

Although this method is simple and convenient, it has a drawback that a spatial distribution of the variables in the model is ignored. This drawback presents as a big discrepancy at the peak as seen at the Fig. 4, and sometimes it brings even worse results than simulations without updating.

In the case of the results from Event 2, as seen at the Fig. 5, there is not much difference between the results with updating and without updating. Steady-state method does not give much effect to the updating of state variables on the Event 2.

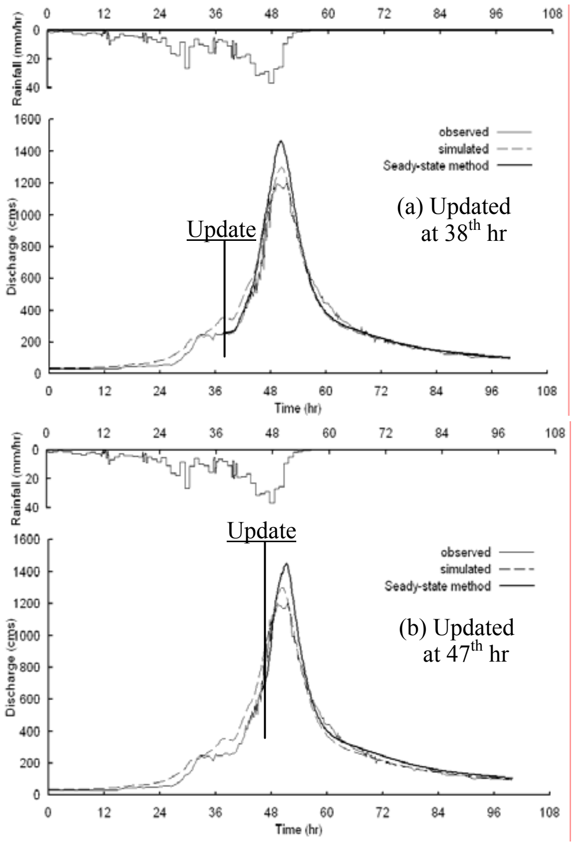

Fig. 4 Update results from steady - state method (Event1).

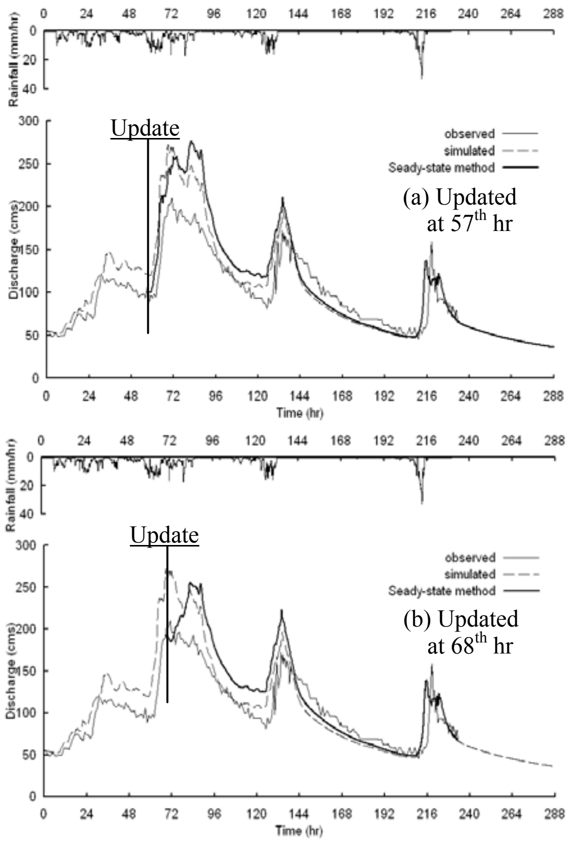

Fig. 5 Update results from steady-state method (Event2).

The different results from two events can be explained by accumulative rainfall amount and average rainfall intensity until the updating point. In the case of Event 1, the accumulated rainfall amount until 38th hr is 231.82mm, which means that the average rainfall intensity is 6.10mm/hr, and 417.98mm until 47th hr (8.89mm/hr). Compared to the high rainfall intensity of Event 1, the accumulated amount of Event 2 is 133.54mm until 57th hr (2.34mm/hr) and 224.18mm until 68th hr (3.30mm/hr). High rainfall intensity may make variable water depth on each point in the basin. And it may be different to the proportionally distributed water depth calculated based on steady-state assumption.

A spatial distribution of the state variables is decided by rainfall distribution and characteristics of the basin, such as area, length, and slope. Therefore there is a need to find a method to update the state variables which considers spatial distribution pattern.

3.2 Ratio method

To contain the spatial distribution of the state variables in the distributed hydrological model, a simulation result right before updating will be used. Under an assumption of similarity, specific ratio calculated from observed discharge and simulated discharge is multiplied to all states variables of each cell.

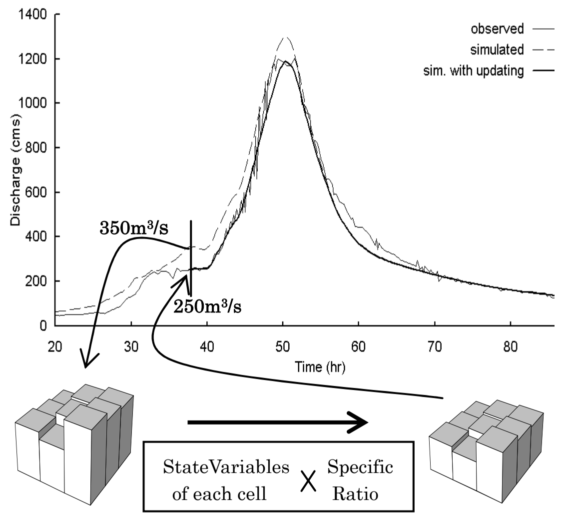

Fig. 6 Updating the storage of each cell with the ratio method.

As for example in Fig.6, the simulated discharge at 38th hr is 350m^3/s and the observed one is 250m^3/s. A specific ratio of observed and simulated discharge is multiplied to all state variables of each cell at this time, and then the simulation continues using the updated state variables. This method is referred to as Ratio method.

Several ratios acquired from observed and simulated discharge are tested for the ratio method. These are a ratio of storage amount and a ratio of discharge itself. Each method will be explained and analysis in the following sections.

(1) Using storage-amount ratio

When we think of a storage amount as a state variable, we can easily come up with a relationship between discharge, Q, and storage, S. The nonlinear Q-S relationship like Eq. (3) is generally used.

$$S(t) = KQ(t)^P \qquad (3)$$

where, S(t) : Storage amount in a basin
Q(t) : Discharge of the outlet
K, P : Constants.

To define the Q-S relationship from the grid-cell based model, steady state with constant rainfall is assumed. After the discharge reach to the steady state, water depths of every cell are stored and converted to the storage amount by multiplying the cell area; cell size in this study is 250m×250m.

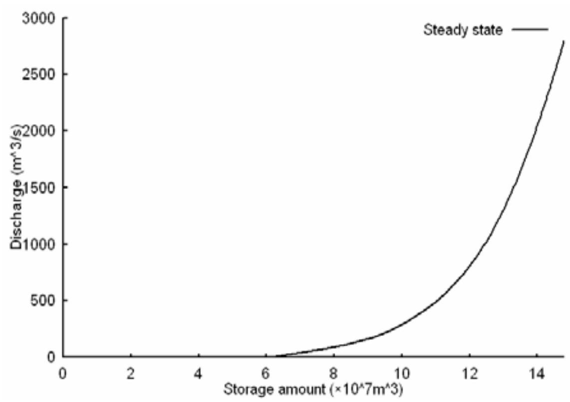

Fig. 7 The Q-S relationship from the grid-cell based model.

From the variable rainfall data, 80 pairs of the Q-S relationship are obtained, and the plot of the relationship is shown at Fig. 7.

The Q-S relationship is used to convert the observed discharge to storage amount. A storage amount of simulated discharge can be acquired directly from water depths of every cell. The ratio is calculated by the Eq. (4).

$$R_S = \frac{S_O}{S_S} \tag{4}$$

where, R_S : Ratio of Storage amount
S_O : S-amount using observed discharge
S_S : S-amount from simulation result.

The R_S is used to update water depths of every cell, and then the simulation starts again using the updated water depths as an initial condition. Hereafter, this method will be referred as S-Ratio method. The discharges, the converted storage amounts, and the ratios of each case are presented in Table 1.

Table 1 Ratio from storage amount

Event	Discharge & Storage-amount			R_S
Event 1 (38th hr)	249.00 / 349.24	→	0.97292E+8 / 1.03476E+8	0.9402
Event 1 (47th hr)	640.00 / 788.69	→	1.15252E+8 / 1.19596E+8	0.9637
Event 2 (57th hr)	100.00 / 121.85	→	0.81944E+8 / 0.86443E+8	0.9480
Event 2 (68th hr)	191.17 / 267.44	→	0.92619E+8 / 0.97295E+8	0.9519

According to the Figs. 8 and 9, which show the results from the S-Ratio method, we can consider that the updating was performed reasonably. Except case (b) of Event 1, every case shows improved match with observed data right after the updating.

The poor updating result in the case (b) of Event 1 is due to observed data error. Because the observed data are converted one from Kamishiiba

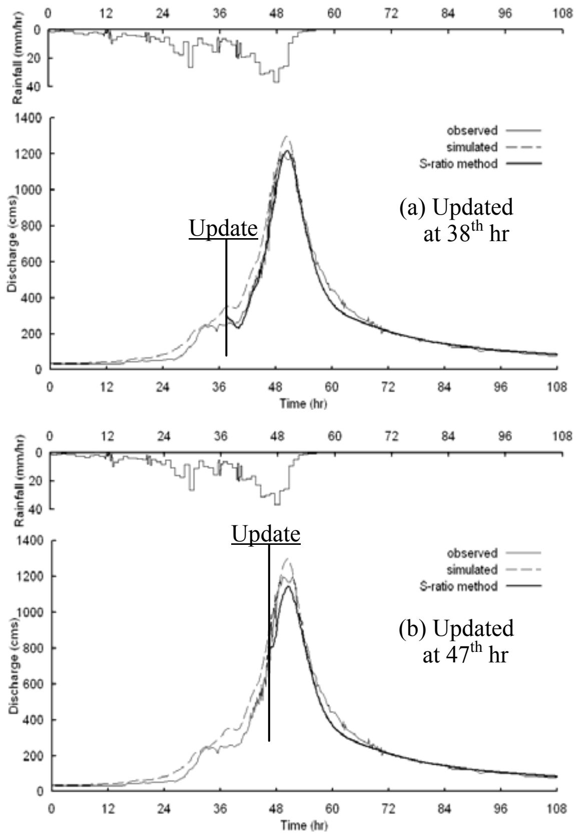

Fig. 8 Update results from S-Ratio method (Event1).

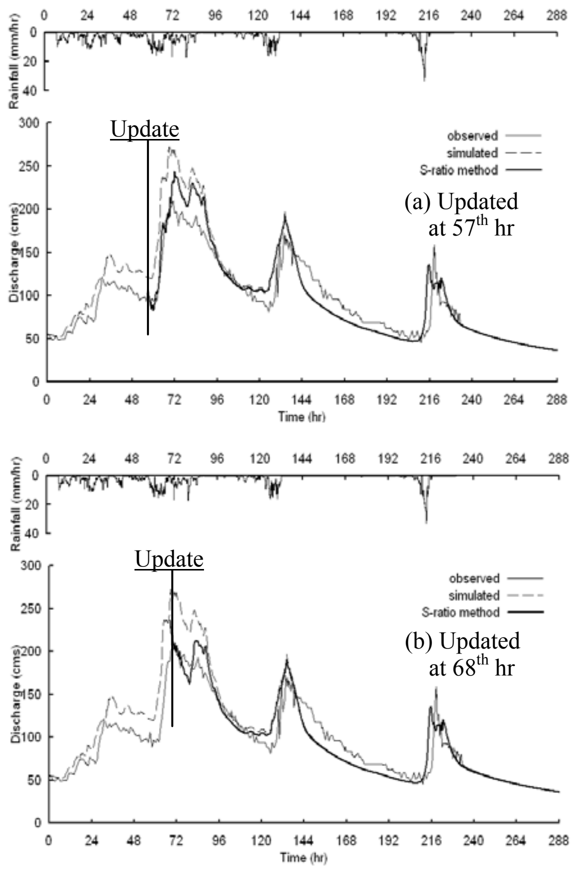

Fig. 9 Update results from S-Ratio method (Event2).

Dam water level, there are 50 to 100m^3/s of oscillations at the sharp rising limb. Because the S_O is directly affected by the observed discharge, the oscillation affect to the value of storage amount at most 1.5 million m^3, and also to the variance of R_S from 0.9759 to 0.9517. For this reason, it is considered that S-Ratio method has vulnerability on the Q-S conversion procedure, especially when the observed data are erroneous.

Also, it should be considered that there is a difference between steady state assumption and unsteady state during simulation. Because the Q-S relationship that is used in the S-Ratio method is acquired from steady state assumption, an unexpected error is already included in the S-Ratio method. Figure 10 shows the different Q-S relationship between steady state and unsteady state during simulation of Event 1.

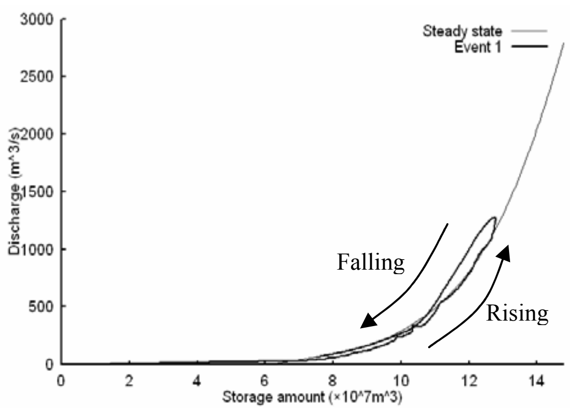

Fig. 10 The Q-S relationship of steady and unsteady state.

(2) Using discharge ratio

The another ratio method is the Q-Ratio method by the ratio of discharge as described in Eq. (5). In the model, every cell has both water depth value and corresponding discharge value to solve the kinematic wave equation.

$$R_Q = \frac{Q_O}{Q_S} \tag{5}$$

where R_Q : Ratio of discharge.
Q_O : Observed discharge at the outlet
Q_S : Simulated discharge at the outlet

It is possible to multiply the ratio to all the discharge of each cell at first, and then convert the discharge to water depth with the stage-discharge relationship of each cell. The Q-ratio method has several merits compare to others-ratio methods as below.

1) Characteristic stage-discharge relationship of each cell can reflect topographic and physical character of cell.
2) Steady state assumption of Q-S relationship is not needed to get an observed storage amount.
3) Very easy to get the ratio and to update.

Table 2 The ratio of discharge

Event	Discharge	R_Q
Event 1 (38th hr)	249.00 / 349.24	0.7130
Event 1 (47th hr)	640.00 / 788.69	0.8115
Event 2 (57th hr)	100.00 / 121.85	0.8207
Event 2 (68th hr)	191.17 / 267.44	0.7148

As seen in Fig. 11 and 12, the state variables are successfully updated with the Q-Ratio method. When the variables are updated properly, better simulation results can be obtained than when the variables are not updated. There are noticeable effects of updating as shown in Fig.11 and case (b) of Fig. 12. In the case (a) of Fig. 12 , the effect is not noticeable because there is no much difference between observed discharge and simulated one at the updating point.

The effect of updating state variables diminish gradually as the simulation goes on. Two reasons are considered for the diminishing. One is because of no parameter changing; the same parameters were used to every simulation in this study. The other one is because of the discharge recession after rain stops. During recession time, state variables may not cause much difference in any methods.

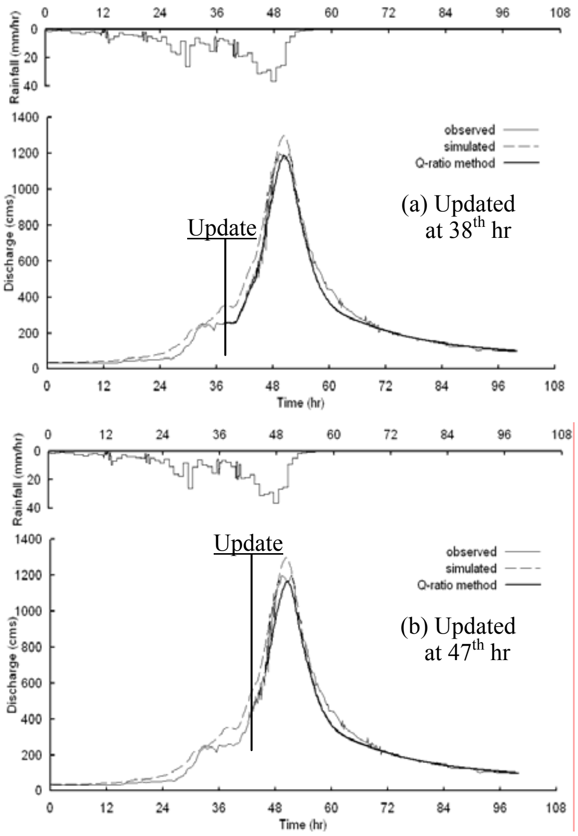

Fig. 11 Update results from Q-Ratio method (Event1).

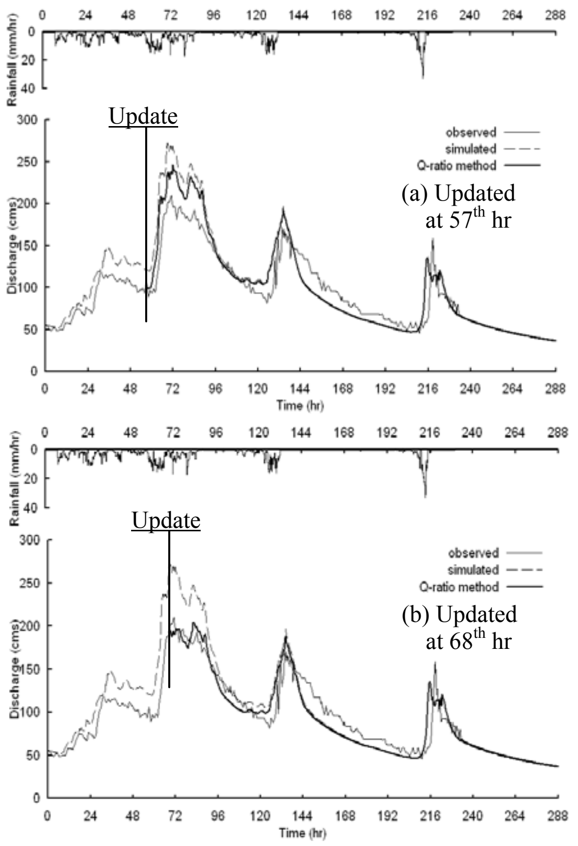

Fig. 12 Update results from Q-Ratio method (Event2).

Results show that updating effectiveness is controlled by an updating point. It seems controlled by the difference between simulated and observed discharges at the updating point and the type of flood event; whether it has one peak or multi-peaks. In practical use, several steps of updating can solve the difficulty of choosing an appropriate updating point.

3.3 Results of comparison

Figures 13 and 14 show the comparison of results from the three ratio methods. When we compare the results, generally Q-ratio method shows good agreement with observed discharge. To check the quantitative updating efficiency, RMSE to the observed data is calculated as Eq. (6).

$$RMSE = \sqrt{\frac{\sum_{i=1,n}(Q_{S,i} - Q_{O,i})^2}{n}} \qquad (6)$$

where, RMSE : Root Mean Square Error
Q_S : Discharge from Ratio method
Q_O : Observed discharge
n : Number of Q_S or Q_O values.

Table 3 shows the RMSE of all time steps after updating. S-Ratio method and Q-Ratio method shows good values compare to the value of Steady-state method.

Table 3 The RMSE – All time step after updating

	Steady-state	S-Ratio	Q-Ratio
Event 1 (a)	62.777	31.801	32.633
Event 1 (b)	63.046	41.740	39.036
Event 2 (a)	37.485	29.703	30.612
Event 2 (b)	36.142	29.152	28.677

Table 4 The RMSE – during 6hrs after updating

	Steady-state	S-Ratio	Q-Ratio
Event 1 (a)	18.132	30.574	11.451
Event 1 (b)	160.941	108.356	90.599
Event 2 (a)	18.376	6.449	17.017
Event 2 (b)	13.409	15.033	10.239

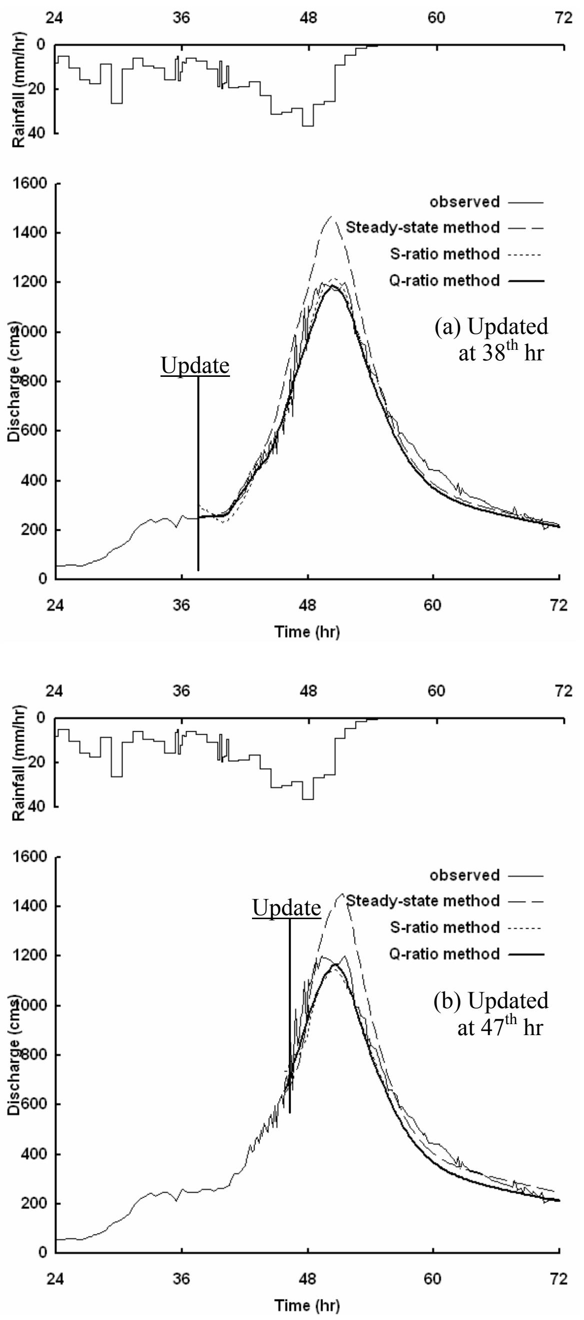

Fig. 13 Update results compare (Event1).

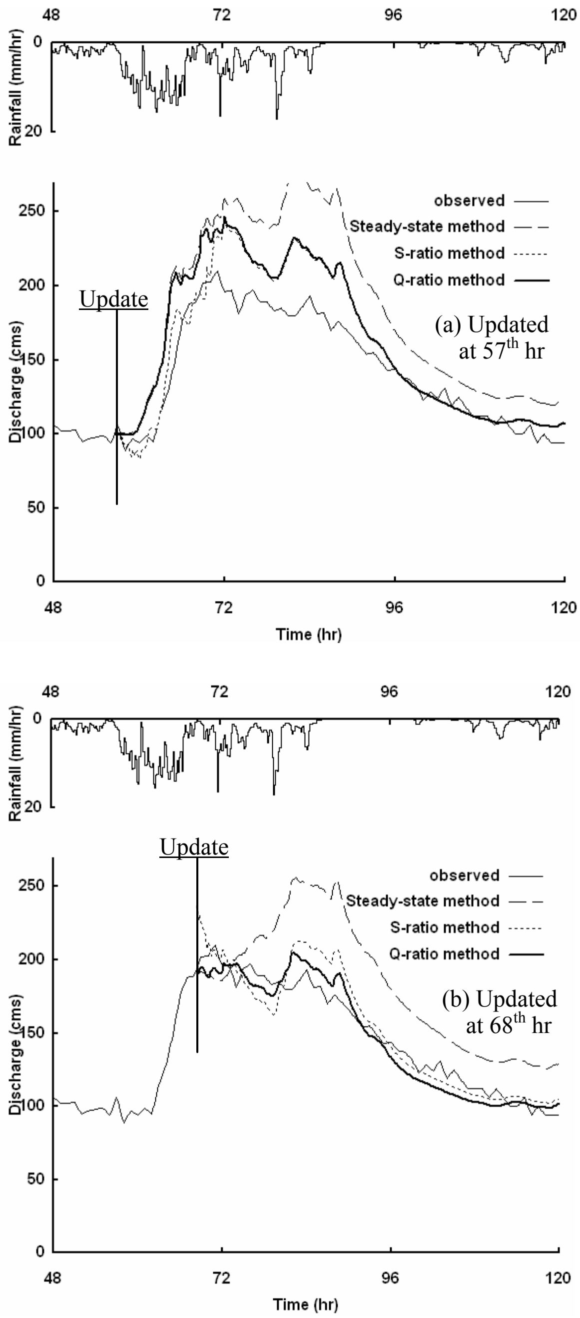

Fig. 14 Update results compare (Event2).

According to the Table 4, the RMSE values during 6hrs after updating present that the Q-Ratio method is the best method to update the state variables in the distributed hydrological model. Furthermore, the Q-Ratio method is the easiest method to update.

4. Discussion and Conclusion

In this study, 3 types of state variables updating methods are tested; Steady-state method and two kinds of ratio method. The pros and cons of each method are as below.

1) Even though the Steady-state method is the simplest way to update the state variables, it is not an appropriate method to update a spatial distribution of the variables in the distributed hydrological model.

2) The S-Ratio method shows improved simulation results after an updating. The Q-S relationship under a steady state assumption is needed

to get a storage amount corresponding to an observed discharge. This assumption may give an unexpected error in the S-Ratio method.

3) The state variables are successfully updated with the Q-Ratio method. Characteristic stage-discharge relationship of each cell can reflect topographic and physical character of cell. Also it is very easy to get the ratio and to update.

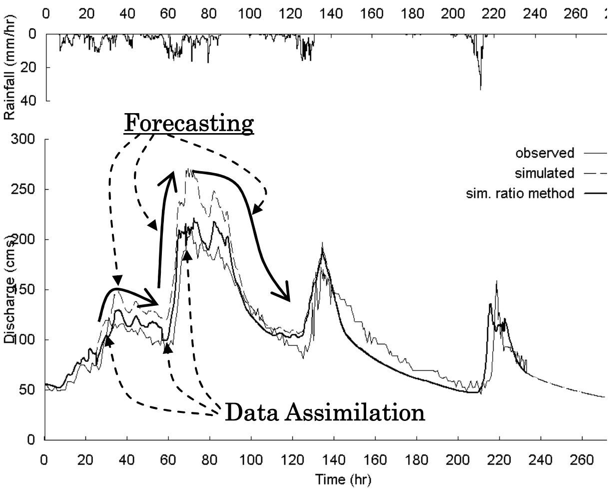

Fig. 15 Expected result from data assimilation.

It is possible to improve real-time forecasting accuracy if the updating method introduced here is conjoined with data assimilation scheme, and if new observed data is available for several steps of updating. Fig. 15 shows improved forecasting accuracy by three steps of Q-Ratio updating with observed data. This case may be regarded as one type of data assimilation when the observed data are believed as accurate.

To carry out appropriate data assimilation, uncertainty of observed data also should be considered, and Kalman filter could be a good method to carry out that. A Kalman filter is an optimal recursive data processing algorithm by combining measured data to prior knowledge about the system, to estimate the state variables for minimizing the error statistically. When applying the Kalman filter theory to the forecasting problem of hydrologic runoff system, there are various treatments as whether the system model is linear or nonlinear, whether the noise is steady or unsteady, or what is taken as the state (Takasao et al., 1989).

Data assimilation with Kalman filter to the grid-cell based distributed runoff model is under way.

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分布型流出モデルにおける状態量の実時間更新

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要　旨

分布型流出モデルを実時間流出予測に用いる場合，空間的に分布する状態量をいかに更新するかが重要な課題となる。本研究では分布型システムにおける流出計算中に状態量を更新する方法をいくつか提案する。それらの方法は流域内に分布する水文量の空間分布を考慮することを可能としている。本研究で提案する方法のうち，一つの方法は，状態量を適切に更新することにより，更新しない場合と比べてよりよい流出シミュレーション結果が得られることを示す。

キーワード: 実時間流出予測，状態更新，分布型流出モデル