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Synopsis

This study proposes a physically-based distributed instantaneous unit hydrograph by adopting the concept of IUH, which is derived based on physical runoff mechanism by combining DEMs, remote sensing and kinematic wave approximation. The heterogeneity of the watershed can be well represented by DEMs, remotely sensed data and kinematic wave approximation in the proposed physically-based distributed instantaneous unit hydrograph. As for ungauged area or the area with poor hydrologic record, the geomorphologic IUH proposed in the study is expected to be a reference for water resources designing and evaluation.

Keywords: remote sensing, DEM, IUH, kinematic wave, heterogeneity

1. Introduction

Physically-based spatially distributed models provide a thorough describing of the watershed hydrologic scheme through its rainfall – runoff simulation. However, massive computation task and CPU time diminish the advantage of this kind of models. On the other hand, conceptual lumped model is criticized for its lacking of physically-based background, while performing very well under the precision of some level.

UH theorem which first proposed by Sherman (1932), by viewing the basin as a linear system and use a hydrologic response function to simulate the relationship between rainfall and runoff, is widely used and accepted tool in hydrologic analysis for many years. However, its linearity and spatial homogeneity assumption about the spatial distribution of rainfall and watershed, makes certain limitation of the model. All watersheds in nature are nonlinear; they are only linear by assumption. Nevertheless, many researchers support that the UH hypothesis is useful (Singh, 1988).

After Sherman’s proposing, several modification of UH theorem has been proposed: Clark (1945) proposed the time-area method. He used the time-area curve of the basin as an input to linear reservoir and obtained the outcome hydrograph as the basin hydrologic response function. Johnston and Cross (1949) modified the time-area curve by including the impact of landscape slope to the water movement, which made the hydrologic response function could reflect the effect of the landscape to the watershed. Edson (1951) assumed the shape of the basin hydrologic response function was a parabola, and simplified the impulse response function to a two parameters characteristic function which related to the watershed geomorphic factors. The Soil Conservation Service (1957) assumed the watershed unit hydrograph as a simple triangle, which simplified the analysis of small scale watershed. Gupta et al. (1969) approach the watershed hydrologic response function in statistic way, that is directly related the peak discharge of the watershed as a function of four geomorphic factors.

Thus, the unit hydrograph theory plays a very important role in the developing of deterministic hydrology. The geomorphologic instantaneous unit hydrograph combined the watershed river network to the estimation process of the hydrologic response function (Rodriguez-Iturbe and Valdes, 1979; Gupta et al., 1980; Lee and Yen, 1997). Lee and Yen (1997) derived the watershed hydrologic response function which could reflect the impact of rainfall intensity to the outflow hydrograph by combining the kinematic wave and GIUH. Among them, Clark’s (1945) work could be seen as the prototype of the instantaneous hydrograph.
IUH based on time-area method concept which derived by viewing the watershed as a distributed nonlinear system is proposed in this study.

There are many time-area methods that appear in the literature, many of them differ only in the method of presentation (Singh, 1988). The central idea of the time-area method is a time contour or an isochrone. If one use orthogonal grid-based data to describe the watershed, an isochrone is a contour joining those grid cells in the watershed that are separated from the outlet by the same travel time. Hence, the time-area diagram indicates the distribution of travel times of different parts of the watershed drains to the outlet (Singh, 1988). Through describing the water movement mechanism by kinematic wave approximation, the travel time to the outlet is calculated; and exploring the relationship between the travel time and the number of grid cells, a histogram (i.e. time-area diagram) is extracted.

The hydrologic response function is viewed as a time-variant characteristic function in the study and is developed to include the effect of water movement mechanism in the output watershed hydrograph. Since the IUH is derived by using kinematic wave approximation, the time base of the IUH is related to the given rainfall intensity. Superposition of different IUHs is adopted when a hydrograph is composed by different rainfall duration.

In this study, a distributed instantaneous unit hydrograph is established and applied to the Yasu River basin (377 km$^2$), which is located in Shiga Prefecture, midwest side of Honshu, Japan. The distributed instantaneous unit hydrograph proposed herein reflects topographic feature (e.g.: land cover, slope, etc.) of the basin in the hydrologic response function, including the water movement scheme in rainfall-runoff processes.

2. The Derivation of the Distributed Instantaneous Unit Hydrograph

Concept of the instantaneous unit hydrograph has been revealed by Clark (1945). His time-area curve is the earliest geomorphologic rainfall runoff model in the world. Model like this needs no hydrologic record but only geomorphologic data to establish the relationship between rainfall and runoff. In this study, kinematic approximation is used to calculate the time of water moves between DEMs grid cells, by dividing the basin into several isochrones, the time-area histogram which denotes the relationship between travel time and area could be derived.

2.1 Usage of DEM

As in Fig. 1, the watershed is divided into several time zones. In each time zone, the time which water drains to the outlet of the watershed is assumed to be the same. Suppose one unit rainfall excess happens on the basin at time step $t = 0$, then the travel time that runoff drains from zone $A_1$ to the outlet is one unit time step, and the volume of the water is $A_1$. Consequently, a time-area histogram could be derived as shown in Fig. 2. This is the time-area method proposed by Clark (1945). By utilizing the DEM, the distance of each cell to the outlet of the basin according to the flow path could be easily obtained.

The DEMs data used in the study is 50m grid digital map, published by Geographical Survey Institute, Japan, at year 1997. The DEM algorithm using in this study is based on the algorithm proposed by Jenson and Domingue (1988) and Jenson and Trautwein (1987). The main idea is by comparing the elevation of the central grid cell to its eight adjacent cells, the flow direction of the central grid could be determined.

Comparing the elevation of the central grid to its eight adjacent cells, the flow direction of the central grid could be determined. According to the flow direction, the flow path from the specific grid to the outlet of the basin and the drainage area of the grid could be retrieved; the distance of each grid inside the
Fig. 4 Using kinematic wave approximation to calculate the traveling between pixels

basin to the outlet is calculated according to the length of the flow path as shown in Fig. 3. The distance-area curve obtained above is simply space related; no temporal relationship is included. Which means that time variation of the hydrologic process could not be revealed by the curve. To obtain the temporal relationship kinematic wave approximation mentioned previously is used, in this study, to transfer the spatial relationship into a temporal one. Land use data for determining the roughness coefficient and rainfall data for outflow calculation is necessary for this aiming.

2.2 Use of kinematic wave approximation

In order to obtain the temporal relationship of any point inside the basin to the outlet, kinematic wave was used, in this study, to transfer the distance into travel time.

Consider a profile of a single grid cell as shown in Fig. 4, according to rational method assumption, assume a rainfall excess of intensity $I_e$ begins instantaneously and continuous indefinitely, the rate of runoff will increase until the time of concentration, when the entire watershed is contributing the flow at the outlet. At and after this moment, the outflow $Q$ can be expressed by the product of rainfall intensity $I_e$ and drainage area $A$.

For a given rainfall excess intensity $i_e$, the equilibrium flow discharge for a given drainage area (contribution area) $A$ is equal to $i_eA$. By acquiring the discharge of each grid inside the basin, the distributed instantaneous unit hydrograph of the basin could be retrieved through describing the mechanism of the water movement.

Here, to consider the direct runoff velocity change due to rainfall excess intensity, kinematic wave approximation is included into the extraction of the IUH. With the velocity of water movement from grid to grid and the distance between grids, the travel time of water movement is obtained.

The momentum equation of kinematic wave could expressed in the form as Eq.(1):

$$q = \alpha (y \cdot \cos \theta)^n$$

(1)

Fig. 5 ASTER image, white line delineates watershed boundary

where $q$ denotes discharge per unit width, $a$ and $m$ are constants, $y$ denotes water depth vertical to the datum and $\theta$ denotes the angle between flow direction and the horizon. This reveals the relationship between the velocity and water depth.

By comparison the equation to Manning’s equation, $a$ is expressed in the form as Eq.(2):

$$\alpha = \frac{\sqrt{\sin \theta}}{n}$$

(2)

where $s$ denotes slope and $n$ denotes Manning roughness coefficient. Substitute Eq.(2) into Eq.(1), and transfer the unit width discharge $q$ into water discharge by multiplying rainfall intensity and grid area, which can solved for water depth $y$ as Eq.(3) as below:

$$y = \sec \theta \left( \frac{n}{\sqrt{\sin \theta}} q \right)^{\frac{1}{n}} = \sec \theta \left( \frac{n}{\sqrt{\sin \theta}} \frac{I_e A}{B} \right)^{\frac{1}{n}}$$

(3)

where $q$ denotes discharge per unit width, $A$ denotes drainage area, $I_e$ denotes rainfall excess intensity and $B$ denotes the width of the cell.

By retrieving the water depth of the grid, traveling time of the grid is solved by dividing distance by water movement velocity as shown in Eq. (4)

$$t = \frac{D \cdot \sec \theta}{V}$$

(4)

where $t$ denotes the transition time of water move between grid cells, $D$ denotes the distance between grid cells and $\theta$ denotes the angle of the slope.

2.3 Roughness coefficient determination

The roughness coefficient was determined from ASTER remote sensing image classification (Fig. 5) incorporating with land use data (Fig. 6) while...
Fig. 6 ASTER image and land use data, the white line delineate watershed boundary

Fig. 7 Land cover classification of ASTER image

Table 1 shows the category of the land cover and its percentage inside the basin.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artificial</td>
<td>12.65%</td>
</tr>
<tr>
<td>Paddy field</td>
<td>12.40%</td>
</tr>
<tr>
<td>Other usage</td>
<td>0.02%</td>
</tr>
<tr>
<td>Water</td>
<td>0.04%</td>
</tr>
<tr>
<td>Forest</td>
<td>52.62%</td>
</tr>
<tr>
<td>Wasteland</td>
<td>6.60%</td>
</tr>
<tr>
<td>Grass</td>
<td>2.72%</td>
</tr>
</tbody>
</table>

There is cloud in the ASTER image.

ASTER is the abbreviation of Advanced Spaceborne Thermal Emission and Reflection Radiometer, which is an imaging instrument that is flying on Terra, a satellite launched from Vandenberg Air Force Base in California, USA in 1999 as part of NASA’s Earth Observing System (EOS). ASTER is a high efficiency optical sensor which covers a wide spectral region from the visible to the thermal infrared by 14 spectral bands. A 2nd level processed image is used. Data acquisition date is 8th, Dec. 2003. The ERDAS IMAGE is used as a tool to proceed the atmosphere correction, geometric correction and land cover classification. The land cover is divided into 8 categories: paddy field, forest, artificial object, wasteland, cloud, other usage, water and grass land. Fig. 7 shows the result of classification.

After the procedure of classification, the data area which covered with cloud or heavy haze will be replaced by using land use data. The land use data is acquired through the web page of Ministry of Land, infrastructure and transport, Japan. Time of data acquisition is 1997. Table 1 shows the percentage of each category inside the watershed.

The value of the Manning roughness coefficient was calibrated by several rainfall events.

2.4 Rainfall data

The IUH proposed in this study assumes a uniform distribution of rainfall excess through out the basin. Thus the accurate rainfall excess estimation is essential to the model performance. The rainfall data was collected from four rainfall gauging stations inside the Yasu River basin, they are Yasu, Minakuchi, Kouka and Oogawara, as shown in Fig. 8. The average precipitation was calculated according to the weight of each rainfall station which obtained by using Thiessen polygon method. Table 2 shows the percentage of each rainfall gauging station.

2.5 Utilization of IUH

According to former researches, rainfall event with different rainfall intensity will cause different outflow hydrograph. That is because the overland flow velocity will increase with rainfall intensity. The IUH proposed in this study can well reflects such hydraulics phynomenum. Fig. 9 shows the relationship between rainfall excess intensity to peak magnitude of IUH and time of IUH peak.
Minshall (1960) found that one UH could not adequately define the shape of the hydrograph derived from a storm of unit duration. Consequently, different UHs would be required to represent the watershed runoff response if the rainfall intensity varied. As shown in Fig. 10, use of different IUH and its related rainfall excess intensity to compose the outflow hydrograph is applied in this study.

By applying the tools and data mentioned previously, a physically based watershed instantaneous hydrograph is extracted.

3. Evaluation Criteria

Four performance evaluation criteria are used to evaluate the performance of the simulation results of model manipulation.

The relative error of peak discharge between simulated and recorded data \(\text{EQ}_p\) is defined as Eq. (5) as below:

\[
\text{EQ}_p(\%) = \frac{(Q_p)_{\text{simulated}} - (Q_p)_{\text{recorded}}}{(Q_p)_{\text{recorded}}} \tag{5}
\]

among it, \((Q_p)_{\text{simulated}}\) and \((Q_p)_{\text{recorded}}\) denote peak discharge of simulated and recorded data respectively.

The relative error of time to peak\((ET_p)\) is defined as Eq. (6) as below:

\[
ET_p \text{ (hr)} = (T_{P\text{simulated}}) - (T_{P\text{recorded}}) \tag{6}
\]

where \((T_{P\text{simulated}})\) and \((T_{P\text{recorded}})\) denote time to peak of simulated and recorded data series respectively.

The coefficient of efficiency \(R^2\) proposed by Nash and Sutcliffe (1970) is defined by the dimensionless expression as Eq. (7):

\[
R^2 = 1 - \frac{\sum [(Q_{\text{recorded}}) - (Q_{\text{simulated}})]^2}{\sum [(Q_{\text{recorded}}) - (\overline{Q_{\text{recorded}}})]^2} \tag{7}
\]

where \(Q_{\text{recorded}}\) and \(Q_{\text{simulated}}\) denote recorded and simulated data series, \(\overline{Q_{\text{recorded}}}\) denote the mean of the recorded data series.

The index of agreement (Lowing and Mein, 1981), IoA, is defined as

\[
\text{IoA} = 1 - \frac{\sum [(Q_{\text{recorded}}) - (Q_{\text{simulated}})]^2}{\sum [(Q_{\text{recorded}}) - (\overline{Q_{\text{recorded}}})]^2} \tag{8}
\]

where \(Q_{\text{recorded}}\) and \(Q_{\text{simulated}}\) have the same definition as above.

4. Results and Conclusions

In this study, three storm events of Yasu River basin were simulated as shown in Figure 11, 12 and 13.

As in Figure 11, the error of peak discharge is 3.3%, and the error of time to peak is 0 hour; As in Figure 12, the error of peak discharge is 1.27%, and the error of time to peak is 2 hour. And as in Figure 13, the error of peak discharge is 6%, and the error of time to peak is 1 hour. The result shows that the instantaneous unit hydrograph simulated the rainfall-runoff relationship of the basin well.
Table 3 Simulation results

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>( EQP ) (%)</td>
<td>3.3%</td>
<td>1.27%</td>
<td>6%</td>
</tr>
<tr>
<td>( ET_P ) (hr)</td>
<td>0 hr</td>
<td>2 hr</td>
<td>1 hr</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.8109</td>
<td>0.9349</td>
<td>0.77</td>
</tr>
<tr>
<td>( IoA )</td>
<td>0.9439</td>
<td>0.9791</td>
<td>0.90</td>
</tr>
</tbody>
</table>

\[
EQP (\%) = \left( \frac{Q_P_{\text{simulated}} - Q_P_{\text{recorded}}}{Q_P_{\text{recorded}}} \right)
\]

\[
ET_P (\text{hr}) = (T_P)_{\text{simulated}} - (T_P)_{\text{recorded}}
\]

\[
R^2 = 1 - \left( \frac{\sum (Q_{\text{recorded}} - Q_{\text{simulated}})^2}{\sum (Q_{\text{recorded}})^2} \right)
\]

\[
IoA = 1.0 - \frac{\sum |Q_{\text{recorded}} - Q_{\text{simulated}}|^2}{\sum |Q_{\text{recorded}}|^2}
\]

The detail of the rainfall events and simulation result is as shown in Table 3.