ON PROPERTIES OF SURFACE WAVES IN AN IMHOMOGENEOUS ELASTIC MEDIUM

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Synopsis

Assuming that the soil ground is composed of a homogeneous isotropic elastic medium, the authors have previously discussed on the dynamic properties of a half-space and a stratum over a rigid medium, and they have also examined the influences of such media on the dynamic responses of structures. It seems almost impossible, however, to investigate the case of a stratified ground having several layers on the basis of the three-dimensional elastic wave propagation theory that have been used in the previous studies. It may then be an interesting approach to this problem that the discontinuous properties of a stratified ground are replaced by some continuous vertical variations of the constitutive parameters, that is, the soil ground is thus presumed as an imhomogeneous elastic medium.

The general solutions may easily be obtained for homogeneous media having horizontal boundary surfaces, because the vector wave equation can always be separated into the longitudinal and transverse components when the two kinds of potentials are introduced. For imhomogeneous media, on the other hand, such method based on the Helmholtz theorem is not successfully applied in general. J.F. Hook have recently presented an generalization of this Helmholtz theorem for some imhomogeneous media whose properties are functions of the single Cartesian coordinate z with constant Poisson's ratio σ. If certain conditions given by a simultaneous nonlinear ordinary differential equation for the constitutive parameters are satisfied, use of this generalization leads to the separation of the vector wave equation.

Making use of Hook's generalization, this paper attempts to investigate the properties of the Rayleigh and Love waves in an imhomogeneous, isotropic, semi-infinite, elastic medium. The constitutive parameters are given by Lamés constants $\lambda/\lambda_0=\mu/\mu_0=(1+z/z_1)^{\gamma/(\gamma-2)}$ and the density $\rho/\rho_0=(1+z/z_1)^{2/(\gamma-2)}$ in which $\gamma=(\lambda+2\mu)/\mu=2(1-\sigma)/(1-2\sigma)$, z_1=an arbitrary constant, and λ_0, μ_0, and ρ_0=the corresponding quantities at $z=0$. The propagation velocities of generalized bodily waves are thus proportional to the square root of the depth, and the present medium is reduced to a homogeneous one when z_1 tends to infinity. It is the only difference in the general solutions that they contain Whittaker's functions with respect to the Cartesian coordinate z instead of the exponential functions for a homogeneous medium.

Numerical results for the frequency equations are summarized as follows:

The dispersion curves, i.e. the relations between the phase velocity and the wave number show an extraordinary similarity between the two kinds of surface waves. They have innumerable modes of propagation all of which are dispersive. All of these modes are separated into the two groups. The phase velocities in one group are monotonically decreasing functions of the product of the wave number and the arbitrary constant z_1, and as the product increases, they approach to the transverse or Rayleigh wave velocity in a limiting homogeneous medium. The modes in the other group generally have the two velocities for an identical wave number, and they vanish with increasing wave number or z_1. The former group is also observed in a stratified homogeneous medium and consequently, it may be concluded that the latter group is inherent in the imhomogeneous medium.