What is Preserved and Lost by a Coarse-Graining Process of Fault Constitutive Law?

〇Reiju NORISUGI, Hiroyuki NODA

Introduction

When constructing a fault constitutive law to model fault slip, an essential step is the coarse-graining (CG) of underlying microscopic processes. In the conventional rate- and state-dependent friction (RSF) law (e.g., Dieterich, 1979; Ruina, 1983), which is widely used to simulate earthquake sequences (e.g., Lapusta et al., 2000), the shear strength is expressed as a function of the "averaged" contact lifetime of a specific spatial scale (see the next Section). However, whether such a coarse-graining is appropriate is unclear. Earthquakes are inherently multiscale phenomena. Seismic slip at very small scales below the CG scale may perturb stress and slip rate at larger scales, via radiated seismic waves. The interaction across scales is not explicitly accounted for in the conventional RSF law, potentially affecting earthquake sequences. Without a plausible CG process or upscaling method, it is not straightforward to apply the same constitutive laws across scales, from rock-friction experiments to megathrust earthquakes. In this study, we summarize the current micromechanical derivation of the RSF law with an explicit CG process at the microscale (Hatano, 2015) and verify its validity through dynamic earthquake sequence simulations based on different CG scales and constitutive parameters.

Rate-state friction from a microscopic perspective

The explanation presented here is based on Hatano (2015), which provides a micromechanical interpretation of the RSF law. An essential point is that the friction is expressed by the summation of shear force sustained by micro-real contacts over the macroscopic CG scale:

$$\mu_{\mathrm{f}} = \frac{1}{N}\sum_{n\in S}\tau_n A_n\,, \tag{1}$$

where μ_{f} is the friction, N is macroscopic normal load, n is the asperity, S is the set of asperities in the CG region, τ_n is the shear stress at the asperity n, and A_n is the asperity area. Under the assumption of uniform slip rate V over S, the RSF law with a single state variable is derived, approximately, as follows:

$$\mu_{\mathrm{f}} \approx \mu_0 + a\ln\left(\frac{V}{V_0}\right) + b\ln\left(\frac{V_0\theta}{L}\right), \tag{2}$$

where

$$\theta = \prod_{n\in S}\theta_n^{\xi_n}, L = \prod_{n\in S}L_n^{\xi_n}, \xi_n = \frac{A_n}{A_{\mathrm{real}}}, \tag{3}$$

and $A_{\mathrm{real}} = \sum_{n\in S} A_n$, and θ_n and L_n are state variables and characteristic state-evolution distances at asperity n, respectively. Therefore, macroscopic θ and L are expressed by the weighted power mean of microscopic θ_n and L_n, and the weight ξ_n reflects the area fraction of real contact n.

Numerical experiment of coarse-graining for a given heterogeneity of state-evolution distance

By using Hatano (2015)'s representation of the RSF law, we can numerically verify whether the CG process is appropriate. However, we found little advantage in using only one state variable so that we extend Eqs. (1-3): The macroscopic shear stress is the average of local shear stress sustained by the nominal area of CG scale. Note that we consider a friction law in the microscopic process, and average it out to yield macroscopic friction law. The distribution of fracture energy G_c is often assumed to be hierarchical, reflecting the fractal nature of fault topography (e.g., Ide & Aochi, 2005). Therefore, we employ the hierarchical distribution of L:

$$r_h = \left(\frac{1}{3}\right)^h r_0,\ N_h = 3^{\lfloor Dh \rfloor} N_0,\ L_h = \left(\frac{1}{3}\right)^h L_0\,, \quad (4)$$

where r_h is the half patch size of hierarchy h, N_h is the number of h-th patches, L_h is the state-evolution distance of h-th patches. Note that the larger h denotes the smaller-scale patches. Then, the macroscopic friction law is rewritten as follows:

$$\mu_f = \mu_0 + a\ln\left(\frac{V}{V_0}\right) + \sum_{h=0}^{H} b_h \ln\left(\frac{V_0\theta_h}{L_h}\right), \qquad \frac{d\theta_h}{dt} = 1 - \frac{V\theta_h}{L_h}, \quad (5)$$

where $b_h = A_h/A_{CG}$, A_h is the area of hierarchy h, A_{CG} is the nominal area of the fault of the CG scale, and H is the number of hierarchy considered. Here, we employ the aging law for state evolution. We coupled Eq. (5) and the elasticity by using the spectral boundary integral equation method (Lapusta et al., 2000) and simulated earthquake sequences. Under different A_{CG}, and L_0, we investigated how the CG process changes the earthquake sequences and what is preserved and lost, to assess the validity of the current CG process.

CG drastically simplifies the earthquake sequences

We found that the CG significantly changes earthquake sequences, so that small ruptures do not occur and ruptures become simpler, with less high-frequency radiation. Figure 1 shows the contour of cumulative displacement on the fault subjected to constant loading from both sides of the fault slipping at V_{pl}. Because large state variables are incorporated into the macroscopic friction when CG is applied, small fragile patches are smeared out, the nucleation size increases, and the small (usually repeating) earthquakes disappear. Results from various parameters and their effect will be discussed in the presentation.

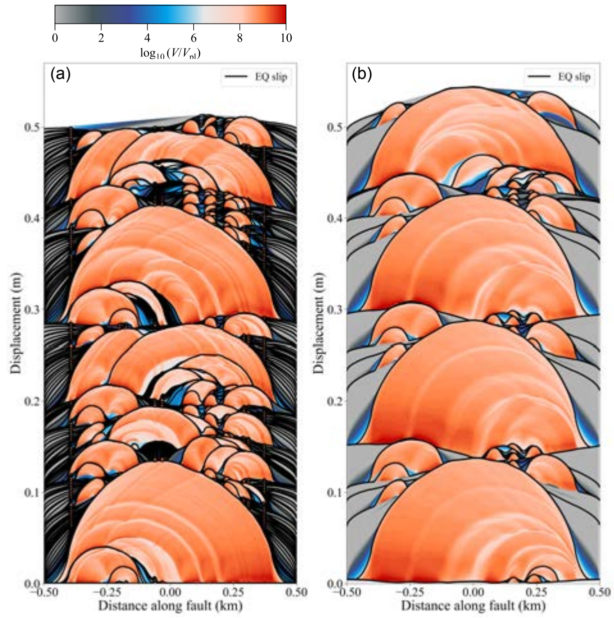

Figure 1: Contour of cumulative displacement color-coded by normalized slip rate. $H = 4$, $L_0 = 1.1$ mm, $N_0 = 1$, and $r_h = 0.5$ km. A_{CG} is $A_0 =$ 1km/8192 in (a) and $2^8 A_0$ in (b), respectively.

Our numerical experiments suggest that the current CG process for constructing the RSF law does not preserve the spatio-temporal complexity of the resultant earthquake sequences. It may be due to an inappropriate assumption that does not mimic seismic behavior at scales below the CG. Investigating whether the constitutive laws, either in a deterministic or stochastic form, exist deserves significant future study.

References

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