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Inconsistency of a Single-Point Evaluation of Traction on a Fault Discretized with Triangular Elements and Several Improved Methods

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Introduction

The boundary integral equation method (BIEM) is a useful tool to model not only crustal deformation caused by subsurface fault slip but also fault dynamics, because it is semianalytic and numerically efficient. Rectangular elements cannot express curved fault with 3-dimentional curvature, and triangular elements are often used for modeling of natural faults. When simulating fault motion, a fault constitutive law is considered, and the traction on the fault. This evaluation requires caution because of the singularity in the elastic Green's function. Previous studies using triangular elements have typically adopted piecewise-constant distributions of displacement gap across faults; however, the discretization details sometimes differ, while the author cannot find the mathematical proof validating these methods. In this study, by considering a trivial problem, a critical issue in the consistency of the triangular mesh was identified, and several methods were proposed resolve it. Here, only static elasticity is considered; however, the outcome is also relevant for elastodynamics as well, as the elastostatic field is left behind after the shear wave radiated from the earthquake source in three-dimensional problems.

Linearly Distributed Gap on an Infinite Fault

Here we consider a trivial problem of linearly distributed displacement gap on an infinite planer fault, which yields zero traction change on it. Such a problem was discretized by a simple triangular mesh defined as Figure 1. Note that the origin O was put at the

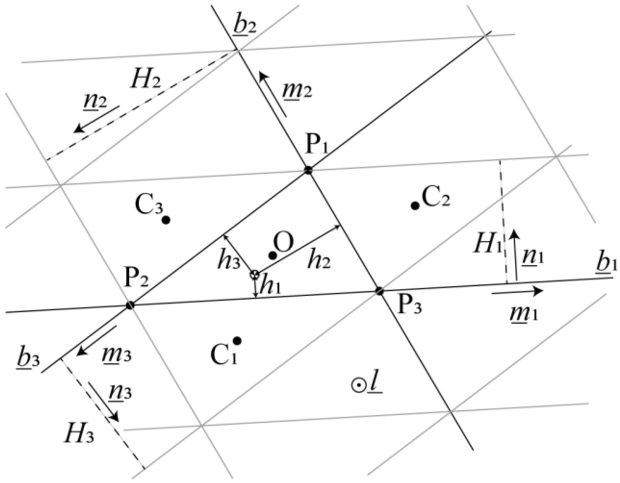

Figure 1. Definition of the simple triangular mesh (adopted from Noda 2025).

centroid of a triangle $P_1P_2P_3$, and C_i is the centroid of neighboring elements. The traction at the evaluation point indicated by a white circle within the triangle $P_1P_2P_3$ can be expressed analytically as

$$\underline{\tau}^{\mathrm{art}} = -\frac{\mu}{2\pi}\underline{\underline{A}}:\sum_{i=1}^{3}\underline{\underline{P}}_i' \, G\left(\frac{H_i}{h_i}\right), \qquad (1)$$

where μ is the shear modulus, $\underline{\underline{A}}$ is the displacement gap gradient,

$$G(\eta) = \pi\cot(\pi\eta^{-1}),$$

and

$$\underline{\underline{P}}_i' = \frac{P_i}{H_i}\left(\underline{m}_i\underline{m}_i + \gamma\underline{n}_i\underline{n}_i + \gamma\underline{l}\,\underline{l}\right). \qquad (2)$$

Because of a geometrical constraint

$$\frac{h_1}{H_1} + \frac{h_2}{H_2} + \frac{h_3}{H_3} = 1,$$

it can be shown that there is no single point in the triangle which gives a correct estimate $\underline{\tau}^{\mathrm{art}} = \underline{0}$. In addition, Eq. (1) predicts that the numerical artifact shows element-wise oscillation whose amplitude is proportional to the gap gradient.

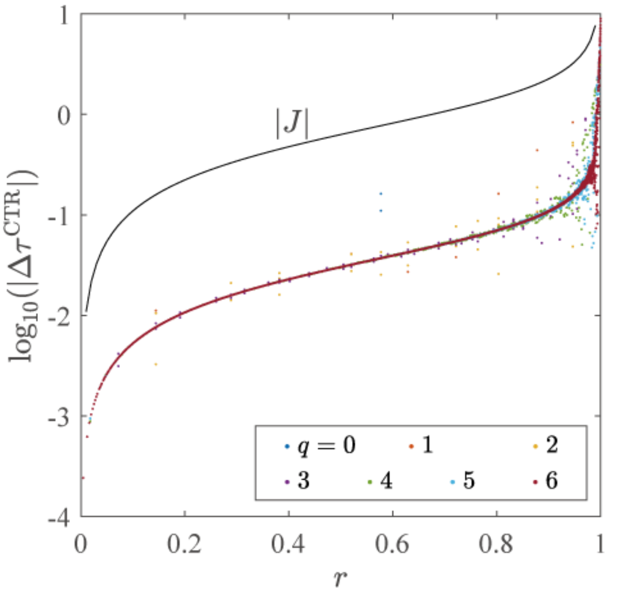

Figure 2. Convergence analysis for the centroid evaluation of traction. $|J|$ represents slip gradient (adopted from Noda 2025).

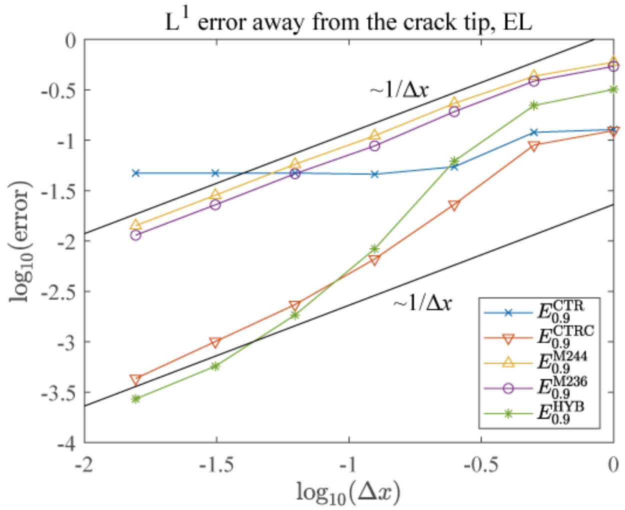

Figure 3. Convergence analyses for various methods (adopted from Noda 2025).

Several Corrected Methods

A convergence analysis was conducted for a problem of a circular crack with uniform stress drop. The analytic solution for slip distribution was discretized, and calculated traction change was compared with the uniform distribution. Figure 2 represents the difference between the analytic and numerical solutions for various resolution. It converged to non-zero function of radius, which is proportional to the slip gradient, as predicted. The L1 error defined away from the crack tip, where singularity exists, did not converge to zero (CTR in Figure 3).

Since we can estimate the artefactual oscillation by Eq. (1), it can be corrected from the centroid evaluation. This method (CTRC) shows convergence and much smaller numerical error than CTR.

In another approach, we can delete the artefactual oscillation by combination of traction estimates at multiple evaluation points. Because the third-order tensor double-dotted with $\underline{\underline{A}}$ in Eq. (1) is a linear combination of three tensors $H_i \underline{\underline{P}}_i'$, which depends solely on the geometry of the mesh, we can cancel $\underline{\tau}^{\mathrm{art}}$ using at least four estimation points. We tested two choices (M244 and M236 in Figure 3), which showed convergence but larger or comparable error relative to CTR. We further linearly combine these two schemes to propose a hybrid method (HYB), which exhibits the best performance among the methods studied here at high resolution.

Discussion

In this study, it was pointed out that the commonly-used discretization with triangular dislocation elements has a critical problem in calculation on on-fault traction. This is the case even of the simplest problem setting of a planer fault with uniform mash for a static problem. Thus, the artefactual oscillation or other numerical error may have affected the accuracy of modeling published so far. It is an important future study if a similar artifact exists or not for discretization of nonplanar fault. Another possible approach is to use a higher-order elements such as a piecewise-linear slip distribution.