P51

A Darcy-Brinkman Formulation for the Hydromechanics of Unsaturated Poroelastic Solids

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Introduction

Fluid flow and transport in porous media can be modeled at different scales. At the continuum scale, the fluid motion in porous media is commonly described by Darcy's law. This law is valid under viscous-dominated creeping flow, where inertial effects are negligible. However, its assumption of uniform velocity and negligible shear make it unsuitable for describing flows with strong velocity gradients or near interfaces. The Darcy-Brinkman (DB) model addresses these limitations by extending Darcy's law through the addition of a viscous diffusion term that accounts for internal viscous shear stresses within the porous medium. In this study, the theory of porous media (TPM)-based multiphase framework by Uzuoka and Borja (2012) for unsaturated poroelastic solids is extended by replacing the conventional Darcy flow law with the more general DB formulation.

Model Description

This study employs an in-house dynamic three-phase coupled finite element (FE) model based on theory of porous media. The numerical framework follows the comprehensive formulation developed by Uzuoka and Borja (2012). The five governing equations for the solid-water-air mixture, modified to incorporate the Brinkman viscous term, are summarized as follows:

(1) Momentum balance equation of overall mixture

$$\rho \boldsymbol{a}^{\mathrm{s}} + \rho^{\mathrm{w}}\left[\frac{D^{\mathrm{s}}\boldsymbol{v}^{\mathrm{ws}}}{Dt} + \{\mathrm{grad}(\boldsymbol{v}^{\mathrm{s}} + \boldsymbol{v}^{\mathrm{ws}})\}\boldsymbol{v}^{\mathrm{ws}}\right] + \rho^{\mathrm{a}}\left[\frac{D^{\mathrm{s}}\boldsymbol{v}^{\mathrm{as}}}{Dt} + \{\mathrm{grad}(\boldsymbol{v}^{\mathrm{s}} + \boldsymbol{v}^{\mathrm{as}})\}\boldsymbol{v}^{\mathrm{as}}\right] = \mathrm{div}(\boldsymbol{\sigma} + ns^{\mathrm{w}}\boldsymbol{\sigma}^{\mathrm{w}}_{\mathrm{vis}}) + \rho \boldsymbol{b}$$

(2) Momentum balance equation of pore water

$$\rho^{\mathrm{wR}}\left[\boldsymbol{a}^{\mathrm{s}} + \frac{D^{\mathrm{s}}\boldsymbol{v}^{\mathrm{ws}}}{Dt} + \{\mathrm{grad}(\boldsymbol{v}^{\mathrm{s}} + \boldsymbol{v}^{\mathrm{ws}})\}\boldsymbol{v}^{\mathrm{ws}}\right] = -\,\mathrm{div}(p^{\mathrm{w}}\boldsymbol{I} - \sigma^{\mathrm{w}}_{\mathrm{vis}}) + \rho^{\mathrm{wR}}\boldsymbol{b} - ns^{\mathrm{w}}\mu^{\mathrm{wR}}(k^{\mathrm{sR}}_{\mathrm{w}})^{-1}\boldsymbol{v}^{\mathrm{ws}}$$

(3) Momentum balance equation of pore air

$$\rho^{\mathrm{aR}}\left[\boldsymbol{a}^{\mathrm{s}} + \frac{D^{\mathrm{s}}\boldsymbol{v}^{\mathrm{as}}}{Dt} + \{\mathrm{grad}(\boldsymbol{v}^{\mathrm{s}} + \boldsymbol{v}^{\mathrm{as}})\}\boldsymbol{v}^{\mathrm{as}}\right] = -\,\mathrm{div}(p^{\mathrm{a}}\boldsymbol{I}) + \rho^{\mathrm{aR}}\boldsymbol{b} - ns^{\mathrm{a}}\mu^{\mathrm{aR}}(k^{\mathrm{sR}}_{\mathrm{a}})^{-1}\boldsymbol{v}^{\mathrm{as}}$$

(4) Mass balance equation of pore water

$$\left(\frac{ns^{\mathrm{w}}\rho^{\mathrm{wR}}}{K^{\mathrm{w}}} - n\rho^{\mathrm{wR}}c\right)\frac{D^{\mathrm{s}}p^{\mathrm{w}}}{Dt} + n\rho^{\mathrm{wR}}c\frac{D^{\mathrm{s}}p^{\mathrm{a}}}{Dt} + s^{\mathrm{w}}\rho^{\mathrm{wR}}\,\mathrm{div}\,\boldsymbol{v}^{\mathrm{s}} + \mathrm{div}(ns^{\mathrm{w}}\rho^{\mathrm{wR}}\boldsymbol{v}^{\mathrm{ws}}) = 0$$

(5) Mass balance equation of pore air

$$\left(\frac{ns^{\mathrm{a}}}{\Theta\bar{R}} - n\rho^{\mathrm{aR}}c\right)\frac{D^{\mathrm{s}}p^{\mathrm{a}}}{Dt} + n\rho^{\mathrm{aR}}c\frac{D^{\mathrm{s}}p^{\mathrm{w}}}{Dt} + s^{\mathrm{a}}\rho^{\mathrm{aR}}\,\mathrm{div}\,\boldsymbol{v}^{\mathrm{s}} + \mathrm{div}(ns^{\mathrm{a}}\rho^{\mathrm{aR}}\boldsymbol{v}^{\mathrm{as}}) = 0$$

Numerical Examples

Three benchmark problems are used to verify the proposed model by comparing the simulated horizontal velocity distributions with analytical solutions. In all cases, the porous medium is assumed to be rigid, with a porosity of 0.4 and a permeability of 1 m/s. No-slip conditions are imposed at the top and the bottom boundaries. The flow is pressure-driven, with an inlet pressure of 1 Pa and an outlet pressure of 0 Pa.

(1) 1D horizontal flow in a saturated porous channel

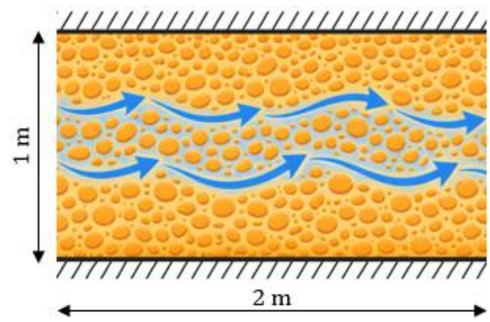

Fig.1. Schematic of the porous domain problem

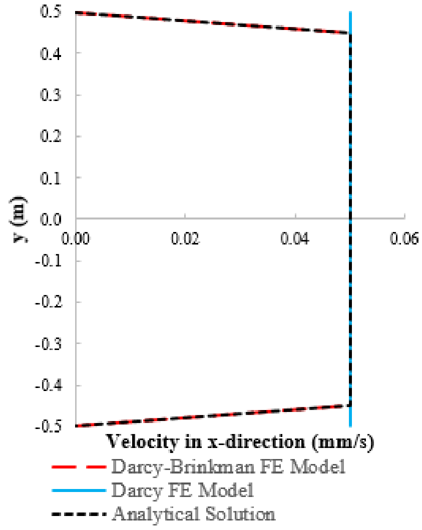

Fig.2. Horizontal velocity profile at mid-channel

(2) Plane Poiseuille flow

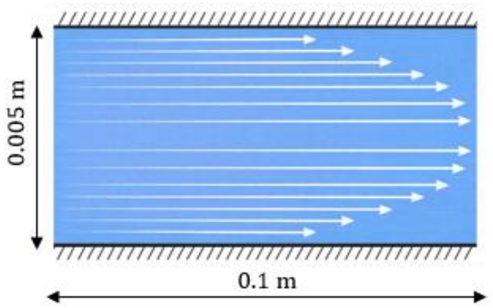

Fig.3. Schematic of the free-fluid domain problem

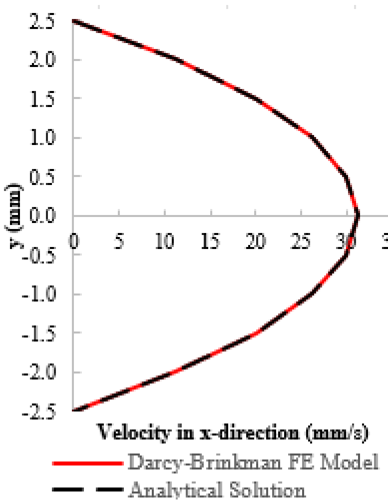

Fig.4. Horizontal velocity profile at mid-channel

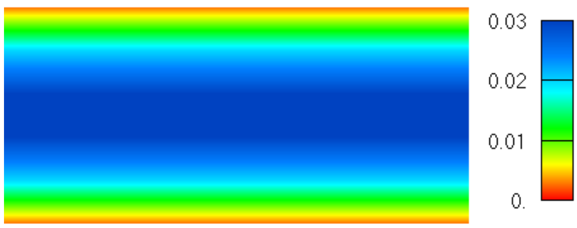

Fig.5. Horizontal velocity in the channel in m/s

(3) Laminar flow over a porous medium

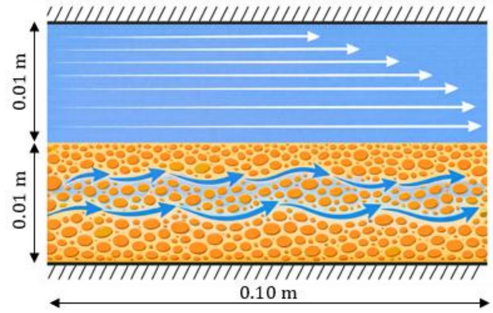

Fig.6. Schematic of the fluid-porous domain problem

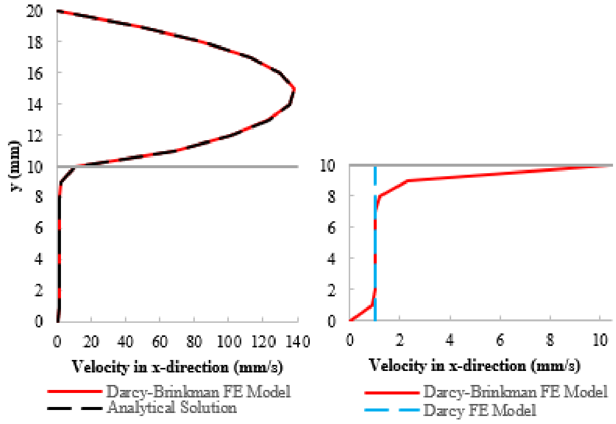

Fig.7. Horizontal velocity profile at mid-channel

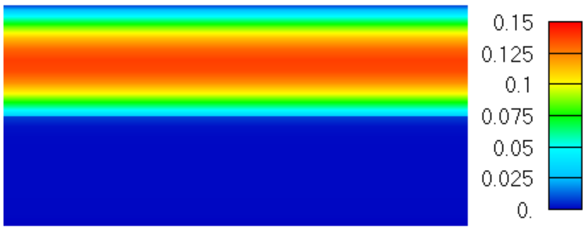

Fig.8. Horizontal velocity in the channel in m/s

Discussion

The simulations show excellent agreement with the analytical solutions, demonstrating the accuracy and reliability of the proposed model, and confirming that the TPM-based formulation correctly reproduces DB flow behavior. The comparison between the two FE models indicates that the DB model is applicable to both porous and free-flow regions, and their transition, providing realistic velocities near walls & at fluid-porous interfaces. In contrast, the Darcy model is restricted to porous media and predicts uniform velocities, failing to capture boundaries and interfacial effects.

References

Uzuoka, R. and Borja, R.I., 2012. Dynamics of unsaturated poroelastic solids at finite strain. *International Journal of Numerical and Analytical Methods in Geomechanics*, 36(13), pp.1535-1573.