Analysis of Design Flood Discharge Allocation in the Yodo River Basin Using Large Ensemble Discharge Datasets and the Multivariate Generalized Pareto Distribution

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This study developed a method based on the multivariate generalized Pareto distribution (mGPD) to estimate the distribution of one variable conditional on extremes of the other. Using discharge data calculated from 9,850 rainfall events extracted from the 720-year d4PDF 5-km dataset, threshold exceedance data were constructed and the mGPD model was fitted to discuss the upstream-downstream balance in Yodo River basin. The results indicate that during 200-year flood events at the Hirakata site, the Kizu River has the highest conditional probability of 150-year flooding (59%) among the three upstream tributaries.

1. Introduction

In river planning, sites and rivers within the same basin are not considered independently. In particular, the discharge capacity of upstream rivers is constrained by the capacity of the downstream reach. However, how such constraints should be quantitatively represented has not been clarified in previous studies.

This study aims to establish a method for characterizing the probabilistic behavior of discharges in upstream rivers conditioned on the downstream site experiencing its design flood discharge. As a case study, the Yodo River basin is selected to demonstrate the proposed methodology.

2. Multivariate generalized Pareto distribution

Exceedances of a random variable over a sufficiently high threshold are asymptotically well described by the generalized Pareto distribution. This property is utilized in the peaks-over-threshold method across a broad range of scientific fields.

Rootzén & Tajvidi (2006)[1] introduced the multivariate extension of the generalized Pareto distribution, mGPD, which is defined as

$$H(\boldsymbol{x}) = \frac{1}{-\log G(\boldsymbol{x})} \log \frac{G(\boldsymbol{x})}{G(\boldsymbol{x} \wedge \boldsymbol{0})} \qquad (1)$$

for G a multivariate extreme value distribution and $\boldsymbol{x}$ an exceedance over threshold $\boldsymbol{u}$, defined as $\boldsymbol{x} = \boldsymbol{z} - \boldsymbol{u}$. $\boldsymbol{z}$ denotes raw discharge values. Based on a pre-specified threshold $\boldsymbol{u}$, these exceedances are extracted and used to fit the mGPD parameters.

In this study, Eq. (1) is examined for the bivariate case, where $\boldsymbol{x} = (x, y)$.

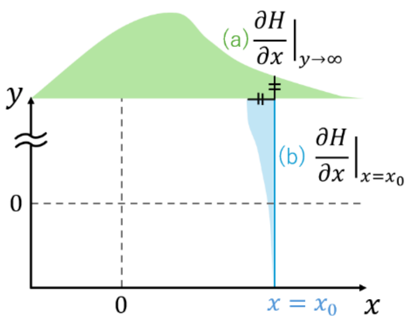

Fig. 1: Conceptual diagram of partial derivative of mGPD with respect to the variable x.

3. Conditional cumulative distribution function

mGPD is a cumulative distribution function. Its partial derivative with respect to x is shown in Fig. 1. The marginal distribution with respect to x can be regarded as a probability density function, as shown in Fig. 1(a). Therefore, for an arbitrary $x > 0$, the partial derivative of the mGPD with respect to x yields the integrated probability density in the y-direction at that value of x, as illustrated in Fig. 1(b).

Since this function does not constitute a cumulative distribution function, it can be normalized to unity, obtaining the conditional cumulative distribution function of y given an arbitrary $x = x_0 > 0$.

$$P(Y \leq y | X = x_0) = \frac{\frac{\partial H(x_0, y)}{\partial x}}{\frac{\partial H(x_0, \infty)}{\partial x}} \quad (2)$$

When the parameters are estimated with x and y denoting downstream and upstream discharge exceedances, respectively, Eq. (2), evaluated at a downstream flood discharge x_0, provides the cumulative distribution of upstream discharges conditional on flooding at the downstream site.

4. Large Ensemble Discharge Datasets

In the process of modeling, mGPD model is fitted to large ensemble discharge datasets. The procedure used to extract the discharge datasets is described below.

(1) From d4PDF-5km rainfall dataset, 9850 rainfall events were extracted.

(2) Rainfall–runoff simulations were conducted for each rainfall event. The simulation period covers the target 24 hours and three days before and after.

(3) For each 7-day rainfall event, the peak hourly discharge at each site was extracted.

The parameters of the 1k-DHM were adopted from Tachikawa et al. (2017)[2).

5. Results

Fig. 2 is the conditional CDF obtained with Hirakata flood discharge x_0, an upper 98% threshold $\boldsymbol{u}$, and a logistic model for the dependence function. Each flood discharge is estimated as an n-year flood discharge using large ensemble discharge datasets, where $n = 200$ for Hirakata site and $n = 150$ for other sites.

Kamo site in Kizu river is flooded in 59% Hirakata flood events, while Hazukashi site in Katsura river and Uji site in Uji river is respectively 17% and 30%. This result shows that among the three upstream rivers, Kizu River is the most dominant contributor to flooding at Hirakata. Alternatively, flooding at Hirakata is often avoided because discharge is reduced by overbank flooding in the upstream Kizu River.

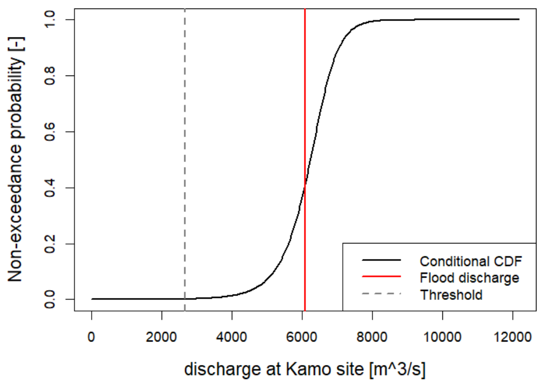

Fig. 2: Discharge distribution at Kamo site conditional on flooding at Hirakata.

6. Conclusion

This study developed a method to construct a conditional CDF given that the other variable takes an extreme value. The proposed framework enables probabilistic assessment of upstream-downstream balance in river planning, where upstream capacity is regulated by downstream discharge capacity. Further investigation is required with respect to threshold selection and dependence modeling.

References

1) ROOTZÉN, H., & TAJVIDI, N. (2006). Multivariate generalized Pareto distributions. Bernoulli, 12(5), 917-930. 2) TACHIKAWA, Y., MIYAWAKI, K., TANAKA, T., YOROZU, K., KATO, M., ICHIKAWA, Y., & KIM, S. (2017). Future change analysis of extreme floods using large ensemble climate simulation data. Journal of Japan Society of Civil Engineers, Ser. B1 (Hydraulic Engineering), 73(3), 77-90.

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