

## A Simplified Method to Estimate the Nonlinear Distribution of Lateral Forces Acting on Stabilizing Piles in Soil Slopes

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**Abstract:** Precise prediction of the lateral force acting on the piles is of significance to assess the stability of pile-reinforced soil slopes. A simplified pressure-based method is presented for estimating the lateral force distribution on the piles embedded in a semi-infinite inclined soil slope. The soil arching theory was incorporated to calculate the driving forces transferred onto piles after determining the active lateral earth pressure between adjacent piles through the horizontal slice method. The published numerical study was selected to examine the applicability of the proposed method. It is demonstrated that the predicted result is in good agreement with the observed data in terms of both the shape and magnitude of the distribution of lateral forces.

### 1 Introduction

For the past decades, the pile stabilization method has come into widespread use to reinforce the slopes which are identified to be potentially unstable. The piles work passively to resist the driving forces of sliding soils and transfer them downward to the underlying stable stratum subjected to the flowing soils. Bearing on sufficient lateral forces, the piles may produce significant internal forces even to failure. In this context, an appropriate estimation of the distribution of lateral forces acting on the piles arising from the sliding mass is therefore necessary for improving the design of piles and slope stabilization. However, limited closed-form analytical solutions have been presented to date regarding the nonlinear distribution of lateral forces resting on the piles.

### 2 Theoretical analysis

#### 2.1 Profile of sliding wedge

Fig. 1(a) shows the profile of sliding wedge. The direction of slip plane at the  $i$ th sublayer can be represented as the line PC based on the theory of pole point (Fig.1b), and the inclination angle  $\theta_i$  is determined as:

$$\theta_i = \angle CPF + \beta = (\angle CIE + \angle EIF)/2 + \beta \quad (1)$$

where  $\angle CIE = \varphi - \beta$ ;  $\angle EIF = \arccos(l_{IE} / l_{IF})$ . According to the geometric relationship of the triangle  $\Delta OIE$  and  $\Delta MIC$ ,  $l_{IE} = l_{OI} \sin \beta$ ,  $l_{IF} = l_{IC} = (l_{OI} + c \cot \varphi) \sin \varphi$ . Eq. (1) is then rewritten as:

$$\theta_i = \frac{1}{2} \left[ \varphi + \beta + \arccos \left( \frac{l_{OI} \sin \beta}{c \cos \varphi + l_{OI} \sin \varphi} \right) \right] \quad (2)$$

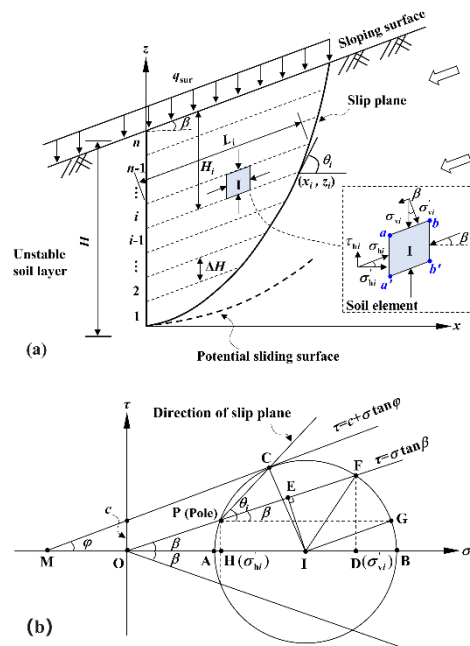


Fig. 1 (a) Profile of cross section UU' with multiple sublayers; (b) the Mohr's circle representation of the stress state on soil element I

To get  $l_{OI}$ , it is found  $l_{IF}^2 = l_{ID}^2 + l_{DF}^2$  in the triangle  $\Delta FID$ . And  $l_{ID} = \sigma_{vi}' - l_{OI}$ ,  $l_{DF} = \sigma_{vi}' \tan \beta$ . Substituting them into Eq. (2) and simplifying, it then follows that:

$$l_{oi} = (A_i - \sqrt{B_i + C_i}) / \cos^2 \varphi \quad (3)$$

$$\text{where } \begin{cases} A_i = \gamma H_i \cos^2 \beta + c \sin \varphi \cos \varphi \\ B_i = \gamma^2 H_i^2 \cos^2 \beta (\cos^2 \beta - \cos^2 \varphi) \\ C_i = c^2 \cos^2 \varphi + c \gamma H_i \cos^2 \beta \sin(2\varphi) \end{cases}$$

## 2.2 Calculation of lateral earth pressure

Fig. 2 shows the force on the  $i$ th sublayer. To determine the active earth pressure  $p_i'$ . The succeeding formulations follow the horizontal and vertical force equilibrium requirements and the moment-balance requirement to the midpoint of the plane  $gh$  with an inclination angle of  $\theta_i$ , and can be expressed as:

$$p_i \Delta H \cos \varphi + c l_i \cos \theta_i - R_i \cos(\pi/2 - \varphi + \theta_i) = 0 \quad (4)$$

$$\begin{aligned} w_i + q_i L_i - c \Delta H - R_i \sin(\pi/2 - \varphi + \theta_i) - c l_i \sin \theta_i \\ - q_{i-1} L_{i-1} - p_i \Delta H \sin \varphi = 0 \end{aligned} \quad (5)$$

$$\begin{aligned} q_{i-1} L_{i-1} (L_{i-1} \cos \beta + l_i \cos \theta_i) / 2 + c \Delta H \cos \beta l_{mi} + \\ p_i \Delta H l_{mi} \sin(\varphi - \beta) - \gamma S_i l_{mi} \cos \beta / 2 - \\ q_i L_i (L_i \cos \beta - l_i \cos \theta_i) / 2 = 0 \end{aligned} \quad (6)$$

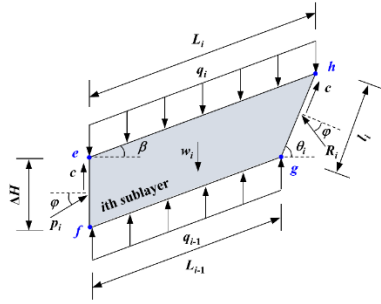


Fig. 2 Force components acting on the  $i$ th sublayer

## 2.3 Soil arching effect behind piles

The stress distribution in the arch zone is shown in Fig. 3. The equation of the radial equilibrium is:

$$d\sigma_r / dr + (\sigma_r - \sigma_\theta) / r = 0 \quad (7)$$

The boundary conditions at the inner and outer boundaries are computed as:

$$\sigma_{in} = \sigma_{r=D_2/2} = C(D_2/2)^{\lambda} (K_p - 1) + 2c\sqrt{K_p} / (1 - K_p) \quad (8-a)$$

$$\sigma_{out} = \sigma_{r=R_{out}} = CR_{out}^{\lambda} (K_p - 1) + 2c\sqrt{K_p} / (1 - K_p) \quad (8-b)$$

As a result, the lateral force per unit depth  $p_{ai}$  transferred onto the piles because of arching, can be calculated by subtracting the lateral force on the soil mass between adjacent piles from the driving force on the outer arch boundary.

$$p_{ai} = \sigma_{out} D_1 - p_i' D_2 \quad (9)$$

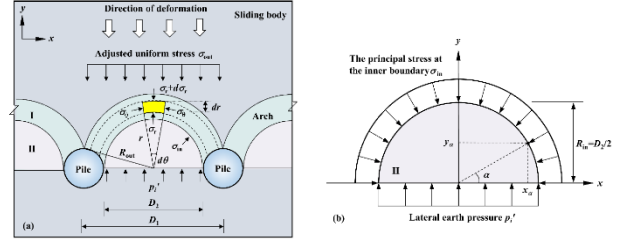


Fig. 3 (a) Schematic representation of the soil arch;  
(b) Uniform stress distribution in zone II

## 3 Method verification

Fig. 4 compared the predicted results using the proposed method with the field test data and numerical results obtained from Lier (2012). It can be seen that The prediction using the proposed method had a satisfactory agreement with the numerical results in terms of the shape and magnitude of distribution of lateral forces. The difference in the predicted maximum lateral forces to the field data was as small as 2.1 kN/m. While the other methods overestimated the lateral forces to different extents.

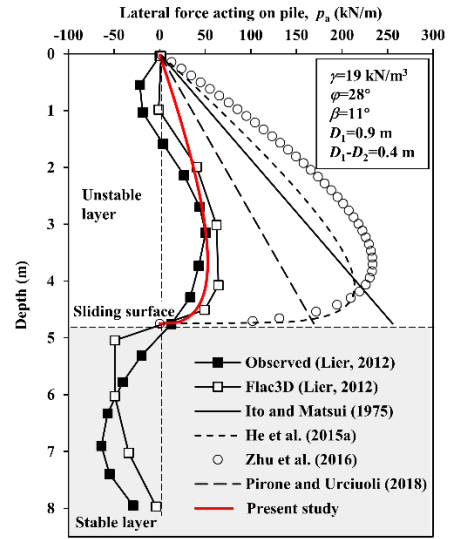


Fig. 4 Comparison of the numerical and calculated results

## 4 Conclusions

A simplified pressure-based method is presented for estimating the lateral force distribution on piles. The predicted results from the proposed method were in good agreement with the observed data in terms of both the shape and magnitude of the distribution of lateral forces.