

Reconstruction of slip distributions on multiple faults with applications to the 2016 Kumamoto Mw7.0 earthquake

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Slip inversion is essential to understand source mechanics of earth quakes. Problems of inverting for static and/or kinematic rupture from surface measurements have been almost always stated to be inherently non-unique in the seismological literature (see e.g. Olson/Apsel 1982; Olson/Anderson 1988; Das /Kostrov 1994; Beresnov 2003; Mai et al. 2016). Statements of this kind mathematically imply that slip/rupture could have never been correctly recovered, because there exist an infinite number of mathematically equivalent solutions to any problem that is inherently non-unique, irrelevant of whether we would have an infinite number of geometrically best possible measurements on any earthquake. Given the geometry of a fault and the displacement functions on the surface of the semi-infinite elastic medium, the slip function on the fault is defined by Fredholm integral equation of the first kind. We prove that Fredholm integral equation of the first kind for slip inversion is mathematically of a unique solution, which is a theoretical guarantee that earthquake slip/rupture can be properly reconstructed if there exist a sufficiently large number of measurements. The non-uniqueness issue of slip inversion in the seismological literature is not mathematically true but is practically caused due to lack of measurements. This practical aspect of non-uniqueness due to lack of data should not be over-emphasized to avoid any potential misunderstanding that reconstructing the slip distribution on a fault could never be possible, physically and mathematically.

We have proposed a new inequality-constrained regularized inversion of slip distributions on multiple

faults. The method implements physically more general inequality constraints to accept more complex dislocation models and/or a combination of complex dislocation models. The formulation of new inequality constraints is theoretically significant in three aspects: (i) it mathematically allows the reconstruction of more complex slip/rupture of an earthquake (if any) from measurements; (ii) it can be applied to a large earthquake involving multiple faults with different rake angles; and (iii) the corresponding new inequality constraints can be thought of as a natural extension of the positivity constraints, or equivalently, the 45 degrees positivity constraints, as first proposed by Olson/Apsel (1982) and Hartzell/Heaton (1983) for a single fault or multiple faults with a single rake angle. The regularization parameter is chosen by minimizing the mean squared errors of the inverted slip solution.

The proposed method is applied to the 2016 Kumamoto Mw7.0 earthquake with GEONET GNSS measurements. We have limited the number of patches to avoid the rank-deficiency (or non-uniqueness) of the linearized version of the Fredholm integral equation of the first kind due to lack of data in the first instant, since a rank-deficient linear system of equations can only produce a set of mathematically equivalent solutions with an infinite number of elements. Although the number of GEONET stations in the study area allows a maximum number of about 180 patches for both Hinagu and Futagawa faults, test computations have shown that with a patch size of 3 km by 3 km, the model is extremely ill-conditioned. Physically reasonable solutions could hardly be expected. Therefore, we have

focused on investigating two patch sizes, namely, one with the size of 4 km by 4 km and the other with the size of 5 km by 5 km. The inequality-constrained regularized inversions with both sizes of patches have resulted in a similar pattern of slip distributions on both Hinagu and Futagawa faults. The ruptures of the earthquake take place shallowly up to 10 km under the surface of Hinagu and Futagawa faults. The inverted slips with the patch size of 5 km by 5 km are preferable in terms of model simplicity and a smaller fitting error, and since there are only a limited number of GEONET displacements larger than 10 cm. The inversion results of slips have shown that:

(i) the 2016 Kumamoto earthquake is only of magnitude Mw6.6-6.7. Nevertheless, if the inequality constraints are not imposed on the slips of Hinagu and Futagawa faults, namely, slips are allowed to move freely around the clock, then the seismic moment magnitude we obtained for the 2016 Kumamoto earthquake is equal to Mw7.13 with a patch size of 4 km by 4 km (or Mw7.16 with a patch size of 5 km by 5 km), which is comparable with what has been reported in the literature, though physically apparently not feasible;

(ii) about 80 to 94 percent of the energy release takes place up to the depth of 10 km or even shallower on Hinagu and Futagawa faults, with the maximum slips of 4.11 to 4.81m on Hinagu fault and 6.93 to 7.89 m on Futagawa fault, respectively, which may well explain the largest damage in Mashiki town observed by Kawase et al. (2017).

Finally, we may also like to note that obtaining a unique solution to a rank-deficient linearized model due to lack of data could not make us closer to the truth of an earthquake, since such a unique solution involves subjective or artificial information. We can never know whether such artificial information is correct or not. Instead, it is highly recommended to use a rougher patch in this case, which may seemingly prevent us from a fine (but arbitrary) imaging of an earthquake; but at the very least, we could get some more or less close-to-true information on the earthquake. Much more effort should be made to advance seismological infrastructures, in particular, innovations of seismometers and space geodesy imaging, for more useful and real-time data on large earthquakes

For more details of this research, please have a look at:

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