

地球物理学的逆問題に対する新しい視点からみる ABIC について  
A new look at Bayesian Akaike's information criterion for geophysical inverse problems

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Akaike's Bayesian information criterion (ABIC) has been widely used in inverse ill-posed problems. Little has been done to investigate its statistical aspects. We present an alternative derivation of the marginal distribution of measurements for ABIC under the assumption of normal distributions and show that the principle of ABIC is to statistically estimate the variances of measurements and prior data by maximizing the marginal distribution of measurements. The determination of the regularization parameter with ABIC is essentially equivalent to estimating the relative weighting between measurements and prior data. We prove that ABIC theoretically would produce a biased estimate of the variance of measurements. Since the prior mean is generally unknown but arbitrarily treated as zero in inverse ill-posed problems, ABIC is shown to fail to produce any reasonable estimate for the prior variance. Although ABIC is constructed under the Bayesian framework, it essentially plays more or less the same role as biased regularization from the frequentist's point of view. ABIC error evaluation cannot be performed under the Bayesian framework but should be more appropriately done with the frequentist's standpoint in terms of mean squared errors. ABIC is sensitive to prior distributions. In the case of non-informative prior distribution, ABIC leads to the conventional weighted least squares (LS) estimate of parameters and cannot be used to solve inverse ill-posed problems. It is not linked to the regularization parameter but only straightforwardly produces an unbiased estimator for the noise level of measurements, which is only applicable numerically for well-posed problems but not for inverse ill-posed problems. Numerical simulated examples are used to demonstrate the statistical performances of ABIC.

More details can be found in:

Xu PL (2021). A new look at Akaike's Bayesian information criterion for inverse ill-posed problems. *Journal of the Franklin Institute* 358 (2021) 4077–4102, <https://doi.org/10.1016/j.jfranklin.2021.03.003>