京都大学防災研究所
Disaster Prevention Research Institute, Kyoto University
- Joint Usage/Collaborative Research Center for Multidisciplinary Disaster Prevention Study -

Ambient Seismic Vibrations in Seismology and Earthquake Engineering

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Outline of Presentation

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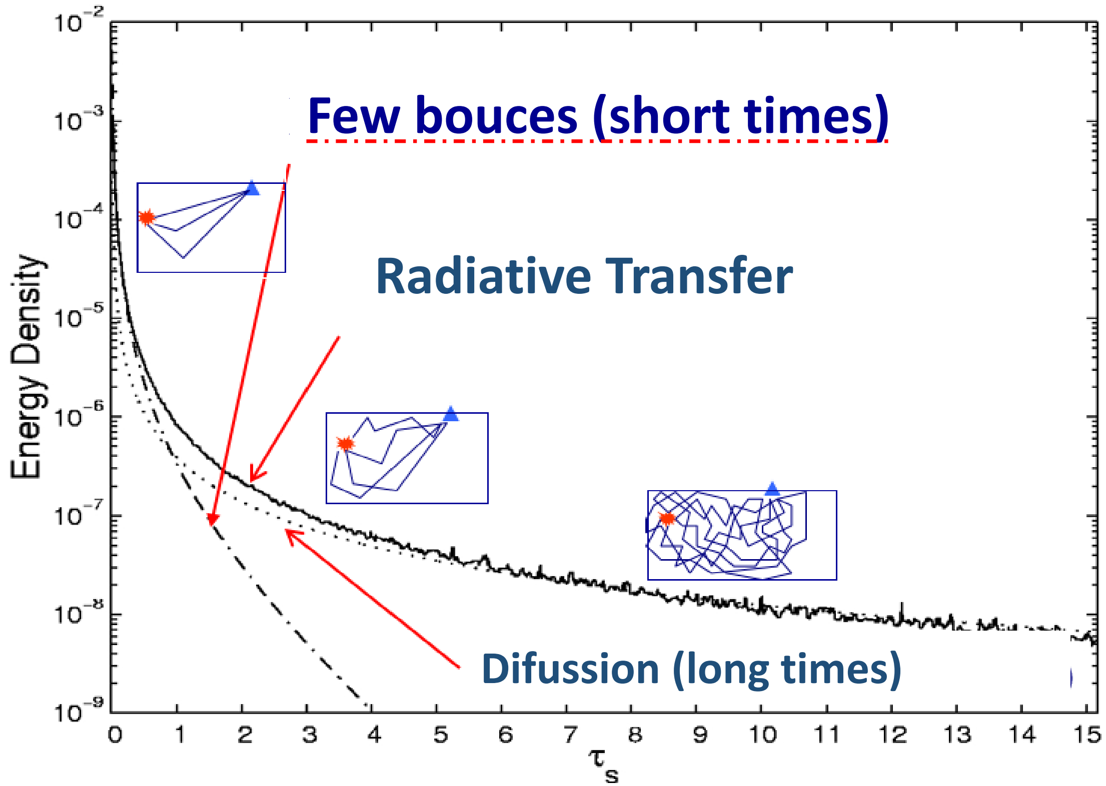
Introduction

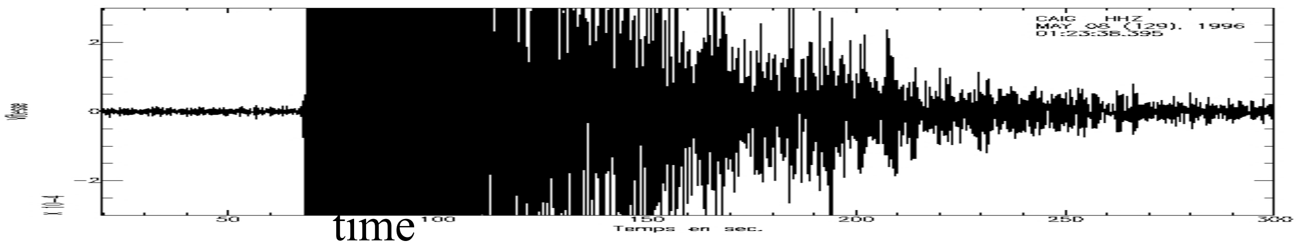
The last decade witnessed the specular emergence of **ambient seismic noise** as a powerful tool for imaging Earth structure at many different scales.

All this lead to improvements of methods that utilize ambient noise. They are now applied to more data sets with better theoretical understanding and are useful for a wide range of applications in **Seismology** including time-dependent imaging for diverse physical processes like helioseismology, underwater acoustics, and structural health monitoring, to cite only a few.

These advances powered also significant applications in **Earthquake Engineering** as well. In addition to improved performance of noise-based imaging, there are innovative applications to establish dominant periods of sites and inverting the soil structure in order to compute seismic response.

Propagation regimes & Energy density decay





Radiative Transfer Theory

Originally in astrophysics by **Chandrasheckar** et al. (40´s & 50's). Introduced to sesmology by **R.S. Wu** (1985) and developend by **K. Aki** and **Y. Zeng**, **H. Sato**, **K. Mayeda**, **M. Campillo**, **L. Margerin**, **A. Gusev** and others.

Sato H. & **M. Fehler** (1998). *Wave propagation and scattering in the heterogeneous Earth*, Academic Press, Cambridge, Mass.

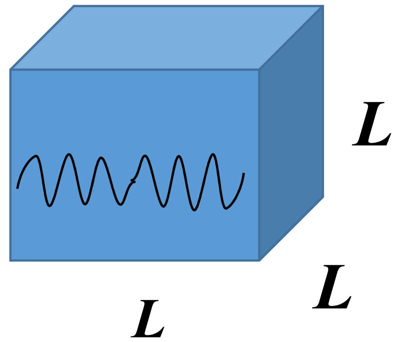
Dmowska R., H. Sato & **M. Fehler** (2008) (Eds) Vol. 50 of *Advances in Geophysics*, Academic Press, Cambridge, Mass.

$$\frac{E_S}{E_P} = \frac{\alpha^2}{\beta^2} \quad \text{in 2D}; \qquad \frac{E_S}{E_P} = \frac{2\alpha^3}{\beta^3} \quad \text{in 3D}$$

Predictions for Elastic Diffuse Fields

Multiple Scattering → Energy Equipartition

Energy Equipartition Principle
Infinite Space, Volume modes



Assume stationary P waves inside a finite region (e.g. A cube of side L >> the wave lenght) and Dirichlet boundary conditions:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = \frac{1}{\alpha^2}\frac{\partial^2 \Phi}{\partial t^2} \qquad \Phi = 0$$

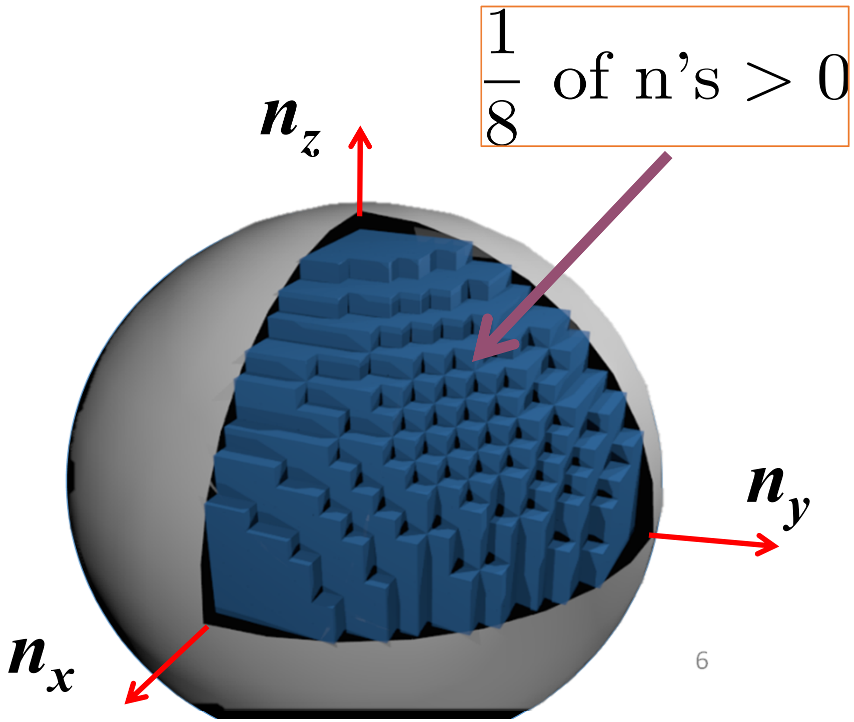
Then we can admit a modal solution of the form,

$$\Phi = A \sin\left(\frac{n_x \pi x}{L}\right) \sin\left(\frac{n_y \pi y}{L}\right) \sin\left(\frac{n_z \pi z}{L}\right) \sin(\omega t)$$

with

$$n_x^2 + n_y^2 + n_z^2 = \left[\frac{L\omega}{\pi\alpha}\right]^2$$

Modal Sphere



According to the Principle of Equipartition of Energy, the energy associated to every state is proportional to the density of modes at a given frequency.

$$N_P = \frac{1}{8} \times \text{Volume of Modal Sphere} = \frac{1}{8} \times \frac{4\pi}{3} \left(\frac{L\omega}{\pi\alpha} \right)^3 = \frac{\pi}{6} (n_x^2 + n_x^2 + n_x^2)^{3/2}$$

$$dN_P = (1/2\pi^2)\omega^2 \alpha^{-3} V d\omega$$

$$dN_S = 2(1/2\pi^2)\omega^2 \beta^{-3} V d\omega,$$

In a diffuse field the energy associated to elastic waves in a volume is arranged according to the **Principle of Equipartition**.

The energy of each state is proportional to the density of modes at a given frequency band.

The ratio of the S and P wave energy is:

$$\frac{E_S}{E_P} = \frac{q \times dN_S}{q \times dN_P} = 2\frac{\alpha^3}{\beta^3} = 2R^3$$

Weaver (1982)

$$\xi_P = \frac{1}{1+2R^3}\xi$$

$$\xi_{SH} = \frac{R^3}{1+2R^3}\xi$$

$$\xi_{SV} = \frac{R^3}{1+2R^3}\xi$$

Weaver (1982)

$$\xi_1 = \frac{1}{3}\xi_P + \frac{1}{6}\xi_{SV} + \frac{1}{2}\xi_{SH} = \frac{1}{3}\xi$$

$$\xi_2 = \frac{1}{3}\xi_P + \frac{1}{6}\xi_{SV} + \frac{1}{2}\xi_{SH} = \frac{1}{3}\xi$$

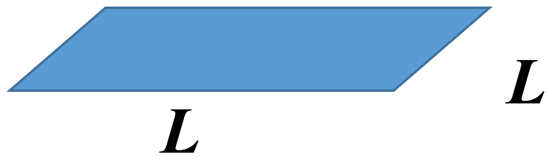
$$\xi_3 = \frac{1}{3}\xi_P + \frac{2}{3}\xi_{SV} = \frac{1}{3}\xi$$

Sánchez-Sesma & Campillo (2006)

Half-Space, Rayleigh's Modes

Assume now stationary Rayleigh waves associated to the free-surface (for instance, a square with side L >> wave length).

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{c_R^2}\frac{\partial^2 w}{\partial t^2}$$

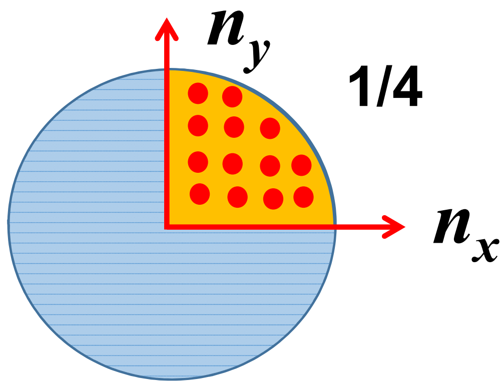


Consider also Dirichlet boundary conditions (w=0 en |x|=|y|=0,L)

$$w = \text{sen}\frac{n_x \pi x}{L} \times \text{sen}\frac{n_y \pi y}{L} \times \text{sen}\,\omega t \rightarrow n_x^2 + n_y^2 = \left[\frac{L\omega}{\pi c_R}\right]^2$$

$$N_R = \frac{\pi}{4}(n_x^2 + n_x^2) = \frac{\pi}{4}\left(\frac{L\omega}{\pi c_R}\right)^2 = \frac{1}{4\pi}\frac{\omega^2 A}{c_R^2}$$

$$dN_R = \frac{1}{2\pi}\frac{\omega A}{c_R^2}d\omega$$



$$E_S = 2\frac{\alpha^3}{\beta^3}E_P = 2R^3 E_P$$

$$E_R = q \times dN_R = \frac{q}{2\pi} \times \frac{\omega A}{c_R^2} d\omega$$

$$E = E_P + E_S = (1 + 2R^3)E_P$$

$$q = \xi_P \times 2\pi^2 \frac{\alpha^3}{\omega^2} \times \frac{1}{d\omega}$$

$$\xi = E/V, \xi_P = E_P/V, \xi_S = E_S/V, \xi_R = E_R/A$$

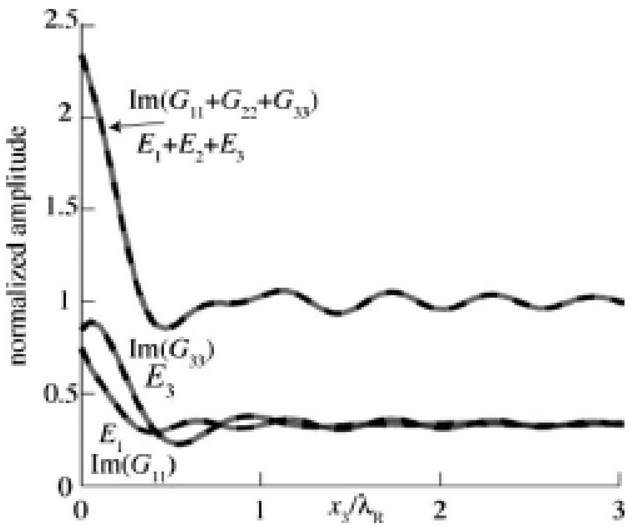
$$\xi_P = \frac{1}{1+2R^3}\xi$$

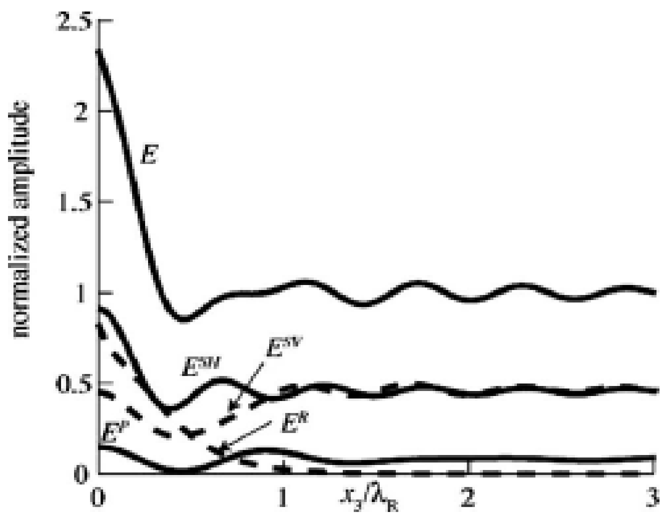
$$\xi_S = \frac{2R^3}{1+2R^3}\xi$$

$$\xi_R = \frac{\pi\beta}{\omega}\left(\frac{\beta}{c_R}\right)^2 \frac{R^3}{1+2R^3}\xi$$

Weaver (1985)
Perton *et al.* (2009)

Semiespacio (3D)





Perton *et al* (2009)

This result shows two ways the equipartition can occur: (1) *à la* Maxwell or (2) *à la* Weaver

Looking for an marker of the diffuse regime

The Equipartition Principle for Elastic Waves
In a diffusive regime all "modes" participate
in the same proportion

$$G_{ij}(\mathbf{X},\xi,t) = \sum_n \varepsilon_n \Phi^n(\mathbf{X}) \exp(-i\Omega_n t)$$

where ε_n are independent random variables

Energy proportions for an infinite elastic solid

$$\frac{E_S}{E_P} = \frac{2\alpha^3}{\beta^3} \quad \text{in 3D}$$

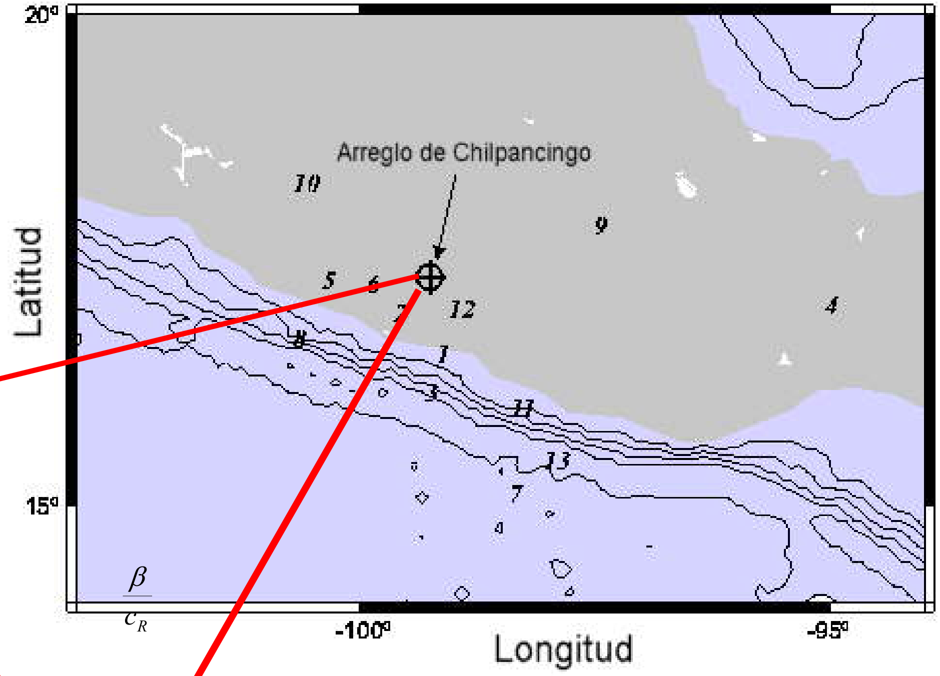
Independent of "scattering" details !

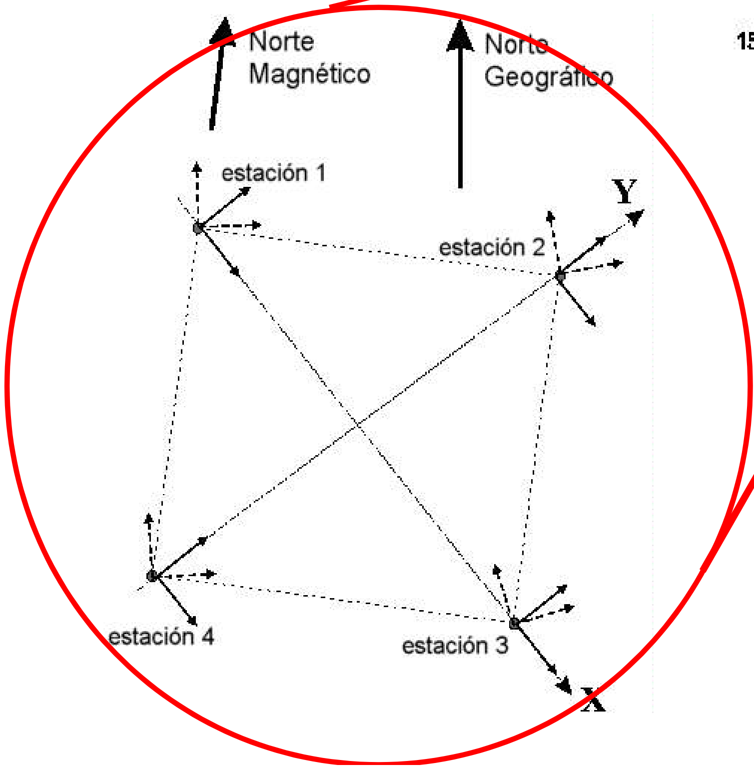
$$\frac{E_S}{E_P} = \frac{\alpha^2}{\beta^2} \quad \text{in 2D}$$

Independent of position in a full-space with homogeneous "illumination" !

The Principle of Equipartition of Energy in Elastodynamics

Experimental Verification

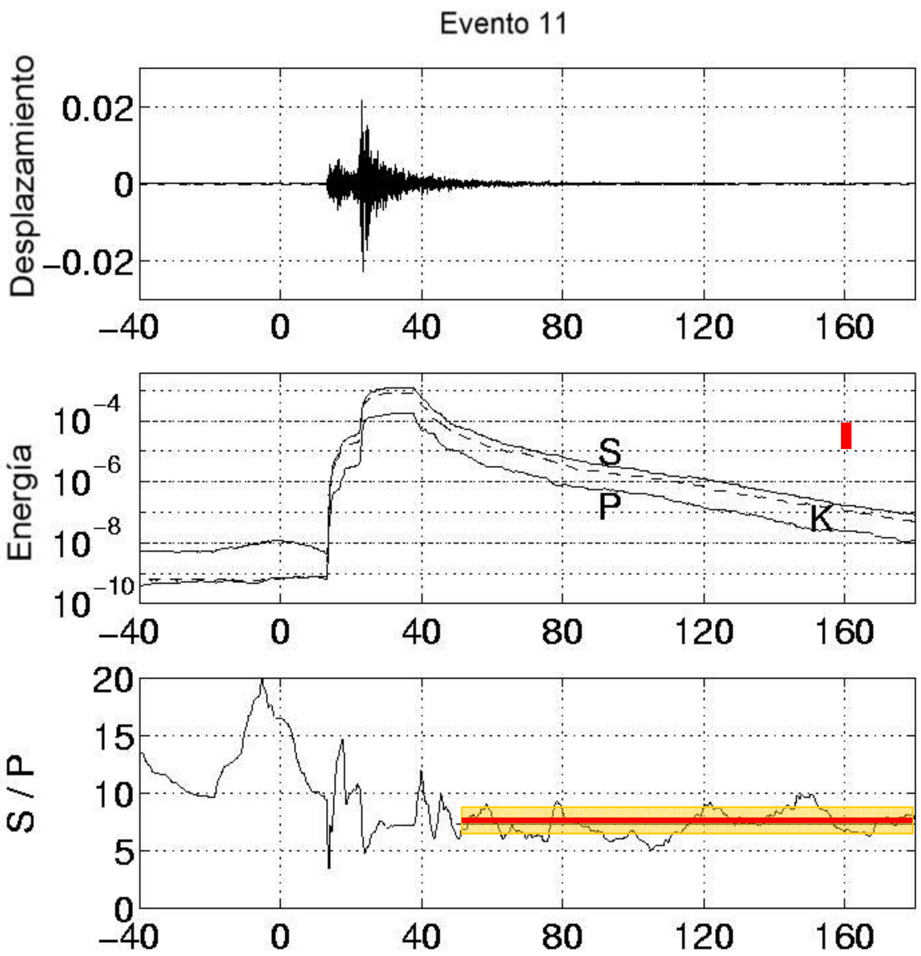


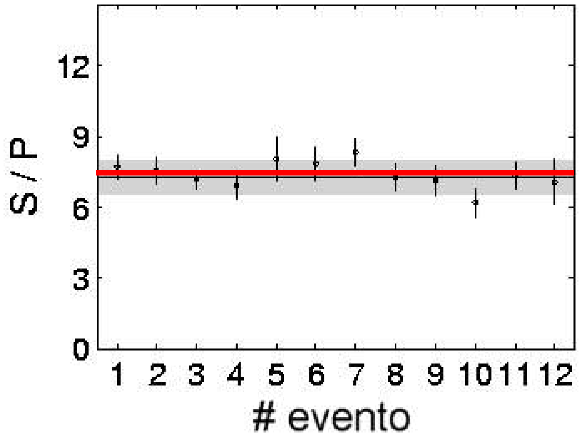


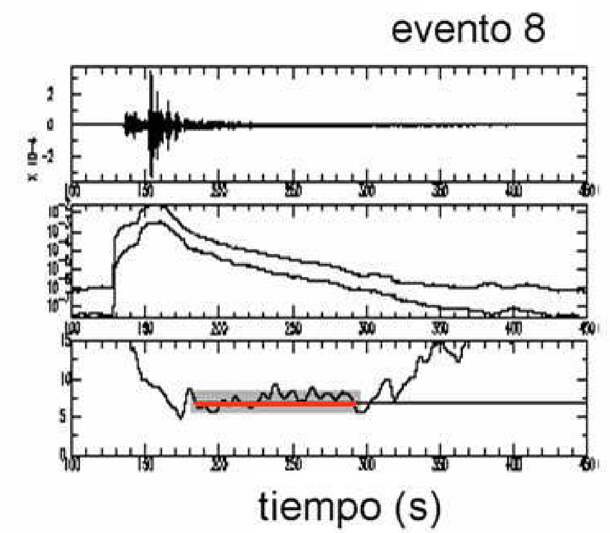
50 m aperture

Campillo et al. (1999); Shapiro et al. (2000)

Experimental Verification







Hennino *et al.* (2001)

Energy ratio	Data $z = 0$	Theory $z = 0$	Theory $z = \infty$	Theory Rayleigh only $z = 0$	Theory Bulk only $z = 0$
S/P	7.30 ± 0.72	7.19	10.39	6.460	9.76
$K/(S + P)$	0.65 ± 0.08	0.534	1	0.268	1.19
$I/(S + P)$	-0.62 ± 0.03	−0.167	0	−1.464	−0.336
H^2/V^2	2.56 ± 0.36	1.774	2	0.464	4.49
X^2/Y^2	0.60 ± 0.20	1	1	1	1

Hennino *et al.* (2001)

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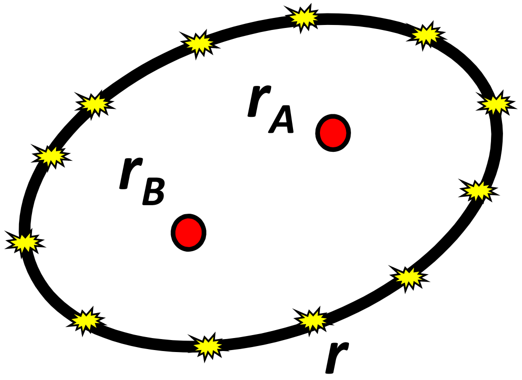
Hennino *et al.* (2001)

Correlation type Representation Theorem

$$2i\,\mathrm{Im}\left[G_{ij}(\mathbf{r}_A,\mathbf{r}_B)\right] = -\oint \left\{G_{il}(\mathbf{r}_A,\mathbf{r})T^*_{lj}(\mathbf{r},\mathbf{r}_B) - G^*_{jl}(\mathbf{r}_B,\mathbf{r})T_{li}(\mathbf{r},\mathbf{r}_A)\right\} dS$$



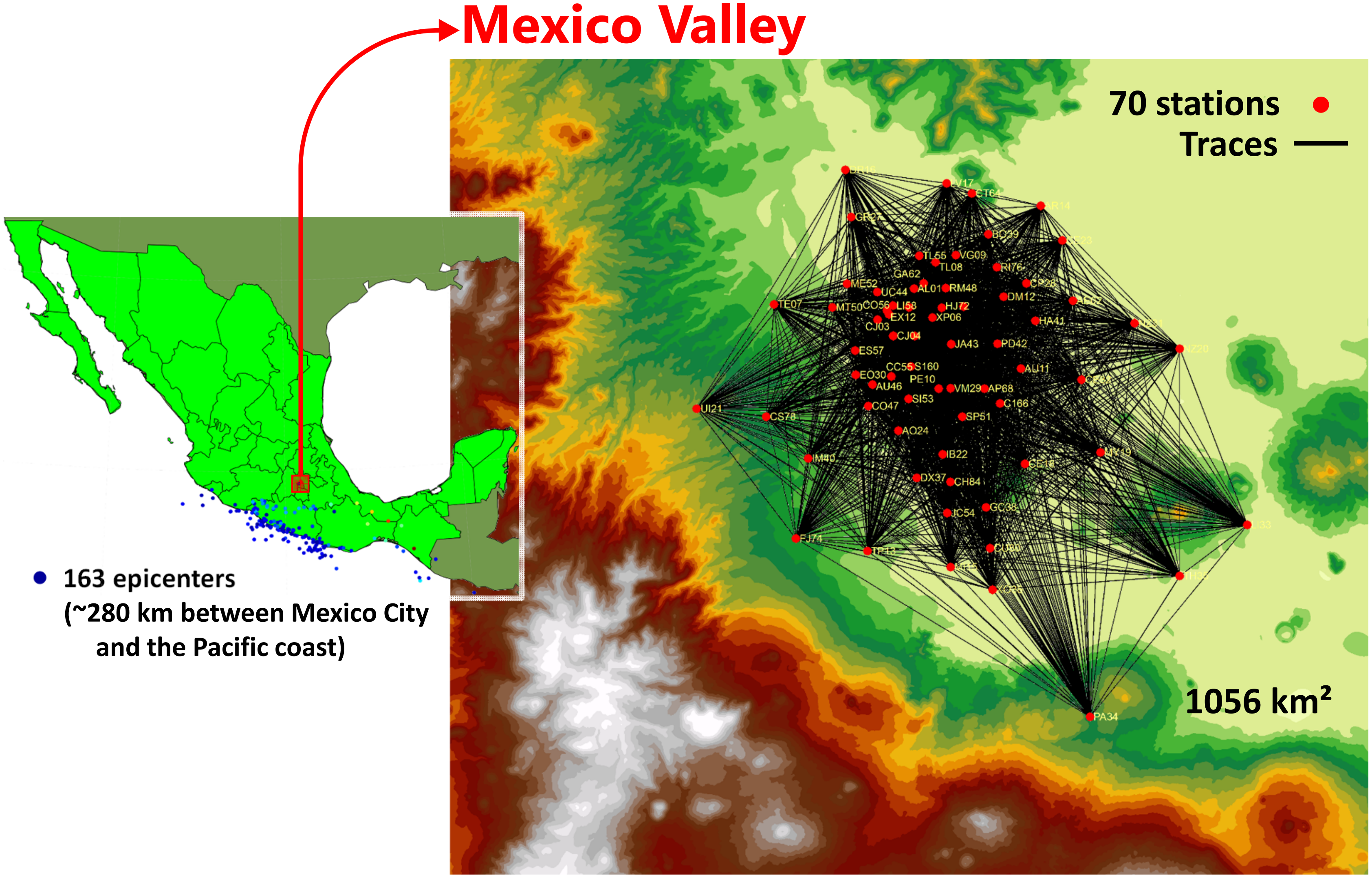
Weaver & Lobkis (2004), Wapenaar (2004), Van Manen, Curtis & Robertson (2006)

$$\frac{E_S}{E_P} = \begin{cases} \dfrac{\alpha^2}{\beta^2} & \text{in } 2D \\ \dfrac{2\alpha^3}{\beta^3} & \text{in } 3D \end{cases}$$

¡ Equipartition !

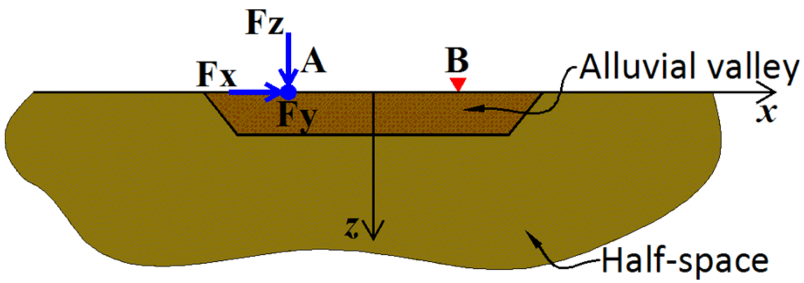
$$\left\langle u_i(\mathbf{x_A},\omega)u^*_j(\mathbf{x_B},\omega)\right\rangle = -4\pi E_S k^{-3}\,\mathrm{Im}\left[G_{ij}(\mathbf{x_A},\mathbf{x_B},\omega)\right]$$

SURFACE WAVE TOMOGRAPHY

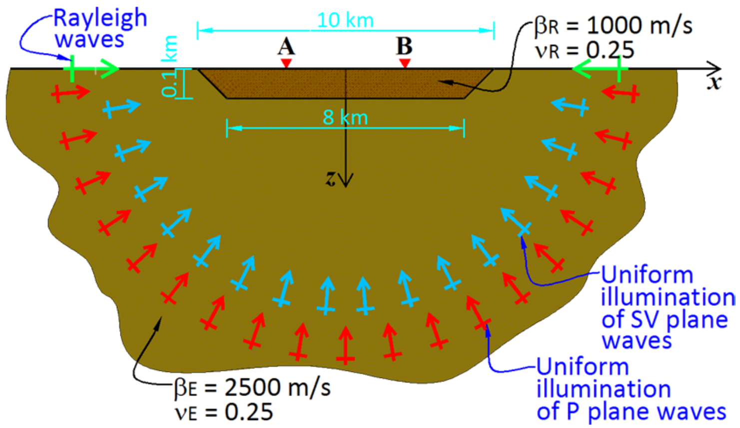


3 WAYS FOR RETRIEVING THE GREEN'S FUNCTION

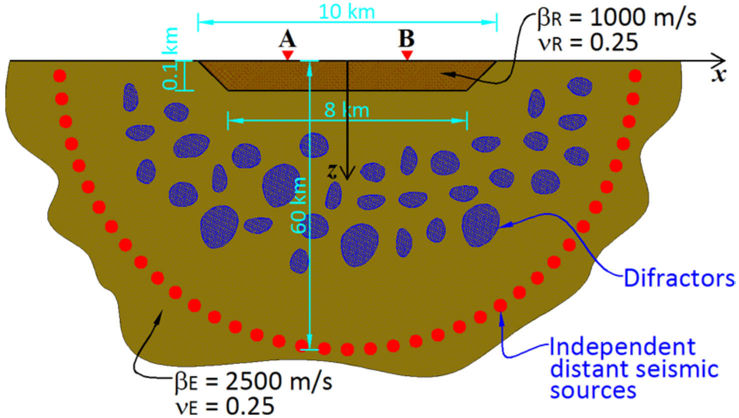
By definition: Impulsive Response



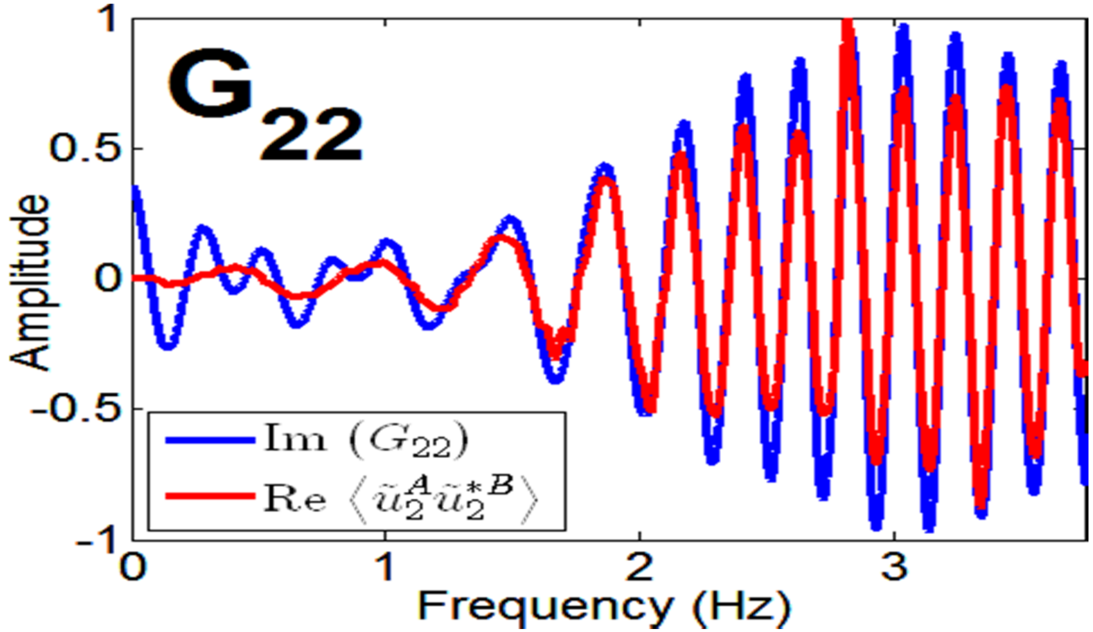
Plane waves: An equipartitioned cocktail of P,SV and Rayleigh waves



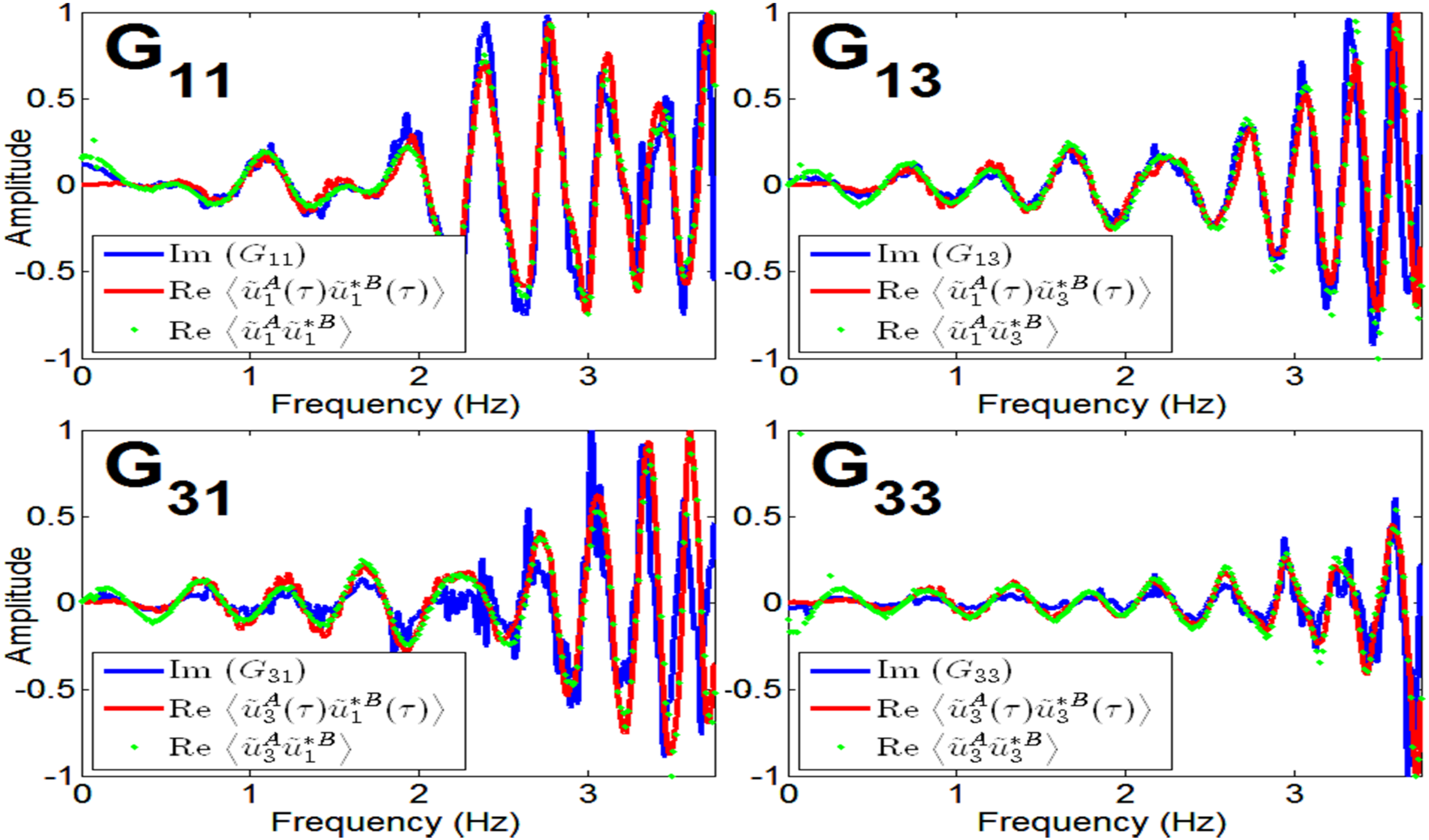
Independent distant seismic sources and lots of randomly placed difractors



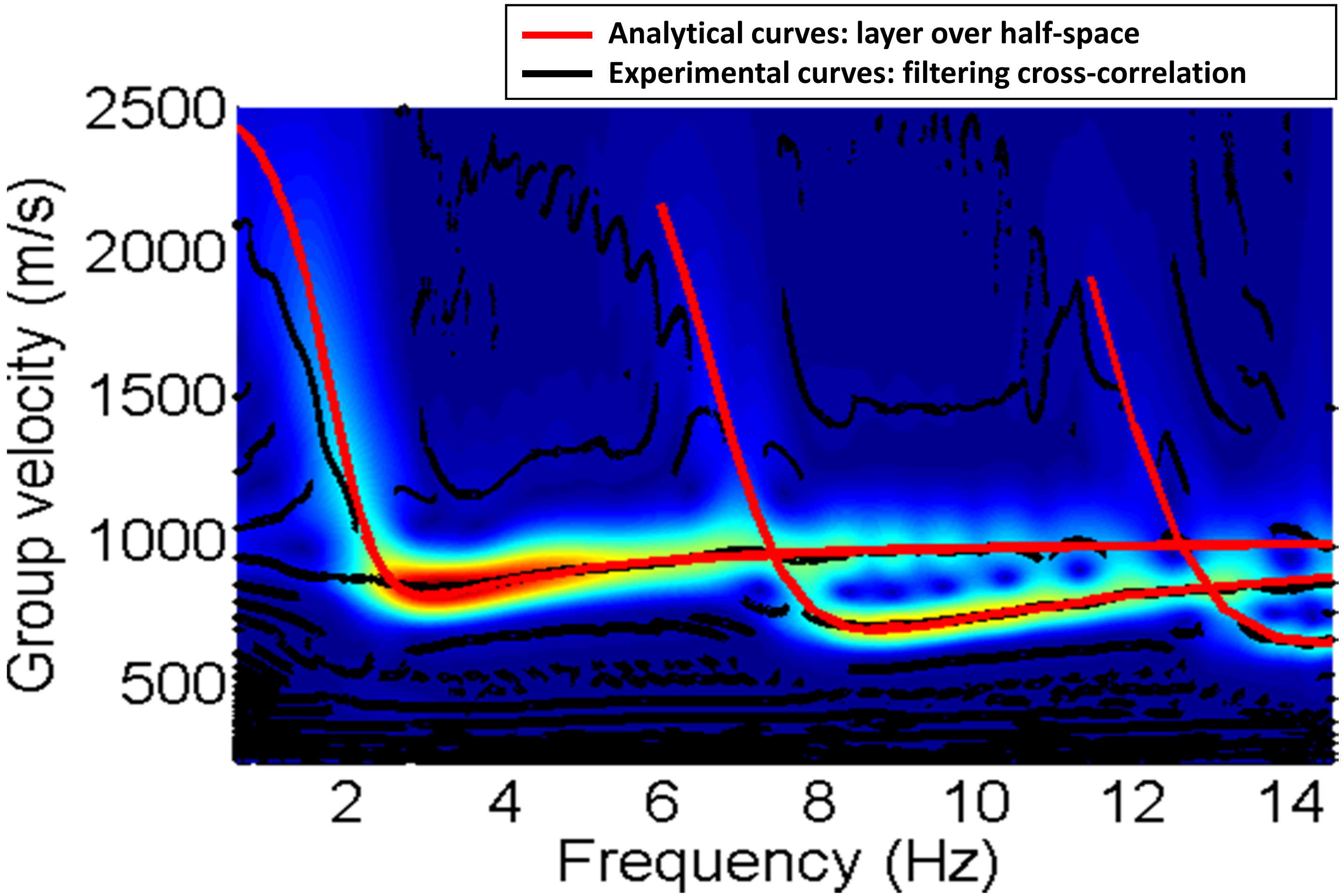
GREEN'S FUNCTION G_{22} (SH CASE)



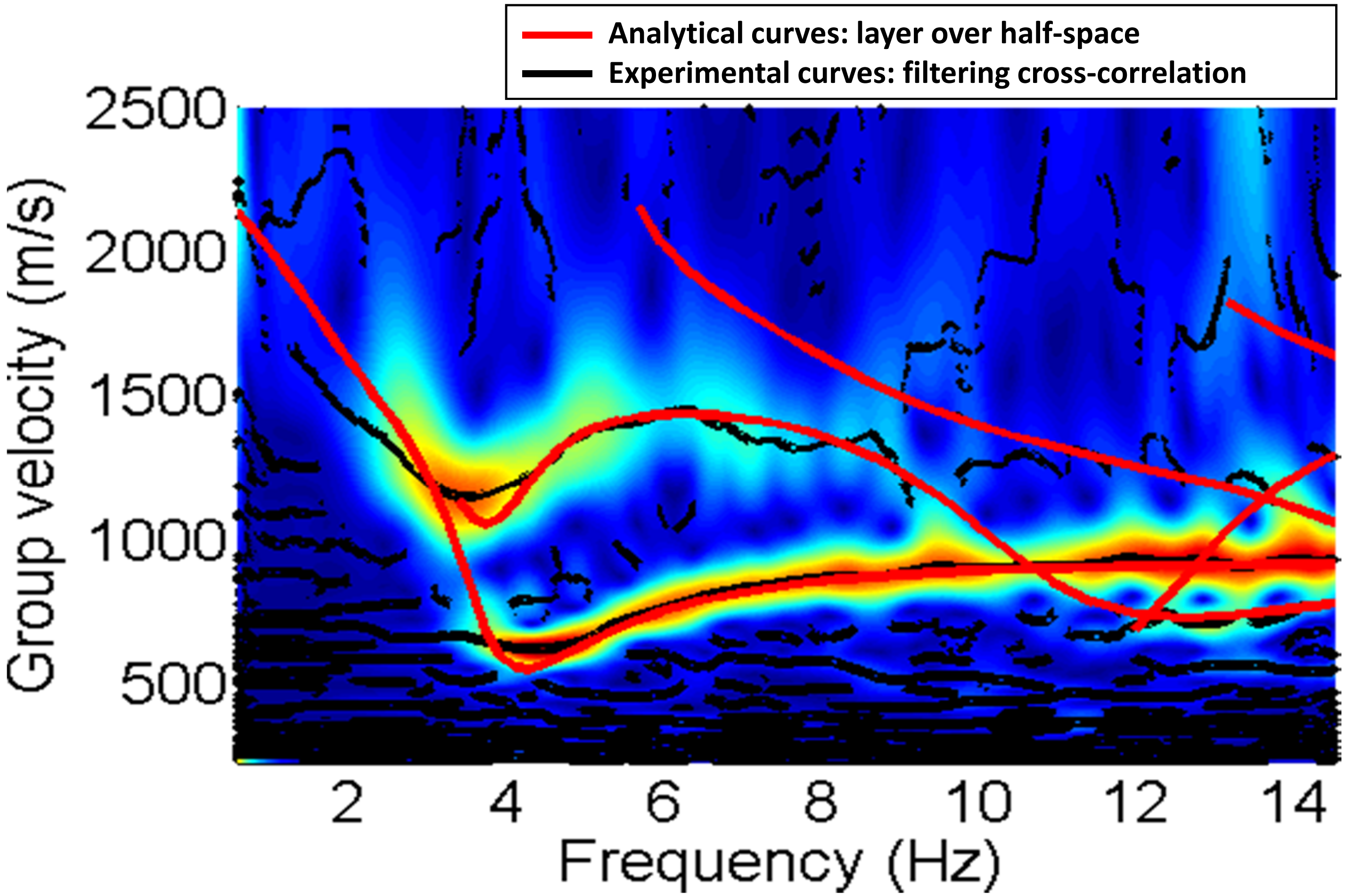
GREEN'S FUNCTION G_{ij} (P-SV CASE)



LOVE WAVES DISPERSION CURVES



Rayleigh waves Dispersion Curves



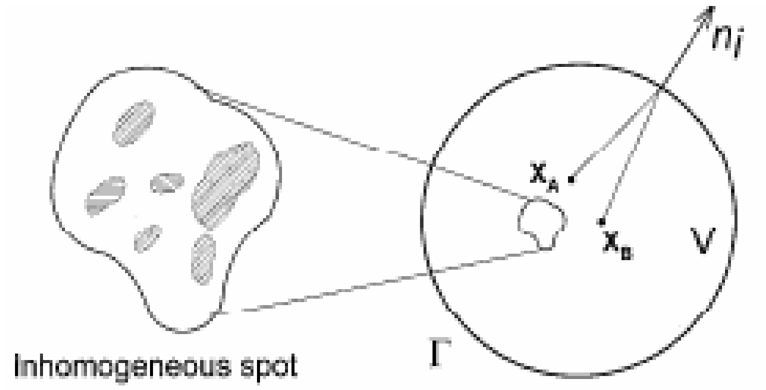
Retrieval of Directional Energy Densities by Averaging Auto-correlations

$$E(\mathbf{x}) = \rho\omega^2 \left\langle u_m(\mathbf{x}) u_m^*(\mathbf{x}) \right\rangle = -4\pi\mu E_S k^{-1} \times \mathrm{Im}[G_{mm}(\mathbf{x},\mathbf{x})]$$

Directional Energy Density (DED). It is the Imaginary part of Green's function at source

$$\mathrm{Re}[G_{11}(\mathbf{x},\mathbf{x};\omega) \times i\omega \left| e^{i\omega t} \right|] = \omega\,\mathrm{Im}\left[G_{11}(\mathbf{x},\mathbf{x};\omega)\right]$$

Proportional to the power transmited to the medium by the unit harmonic force



From the Representation Theorem

$$\mathrm{Im}[G_{mn}(\mathbf{x}_A,\mathbf{x}_B)] = \frac{-\omega}{16\pi^2 r_A \rho}\int_\Gamma \left\{ \frac{\exp(iqr\gamma_j n_j)}{\alpha^3} n_m n_n + \frac{\exp(ikr\gamma_j n_j)}{\beta^3}(\delta_{mn} - n_m n_n) \right\} d\Gamma_\xi$$

$$E_1 = A \times \mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x})] = A \times \frac{-\omega}{12\pi\rho} \times \left\{ \frac{1}{\alpha^3} + \frac{2}{\beta^3} \right\} = E_{1P} + E_{1S}$$

Stokes (1849)

Sánchez-Sesma *et al.* (2008)

AS A CONSEQUENCE OF THE IDENTITY

Energy ⟷ *Green's Function*

$$E_1 + E_2 + E_3 = A \times \mathrm{Im}[G_{kk}(\mathbf{x},\mathbf{x})] = E_P + E_S$$

$$E_1 = \rho\omega^2 \left\langle u_1^2 \right\rangle \propto \mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x})]$$

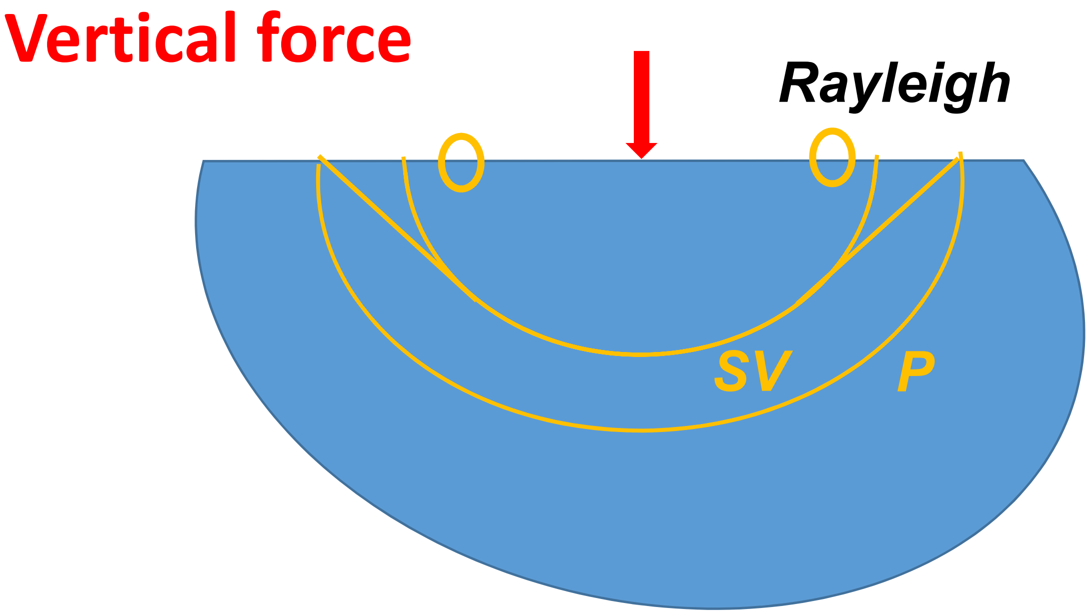
$$E_2 = \rho\omega^2 \left\langle u_2^2 \right\rangle \propto \mathrm{Im}[G_{22}(\mathbf{x},\mathbf{x})]$$

$$E_3 = \rho\omega^2 \left\langle u_3^2 \right\rangle \propto \mathrm{Im}[G_{33}(\mathbf{x},\mathbf{x})]$$

Directional Energy Densities (DEDs)

Perton et al. (2009)

Deterministic Partition of Energy



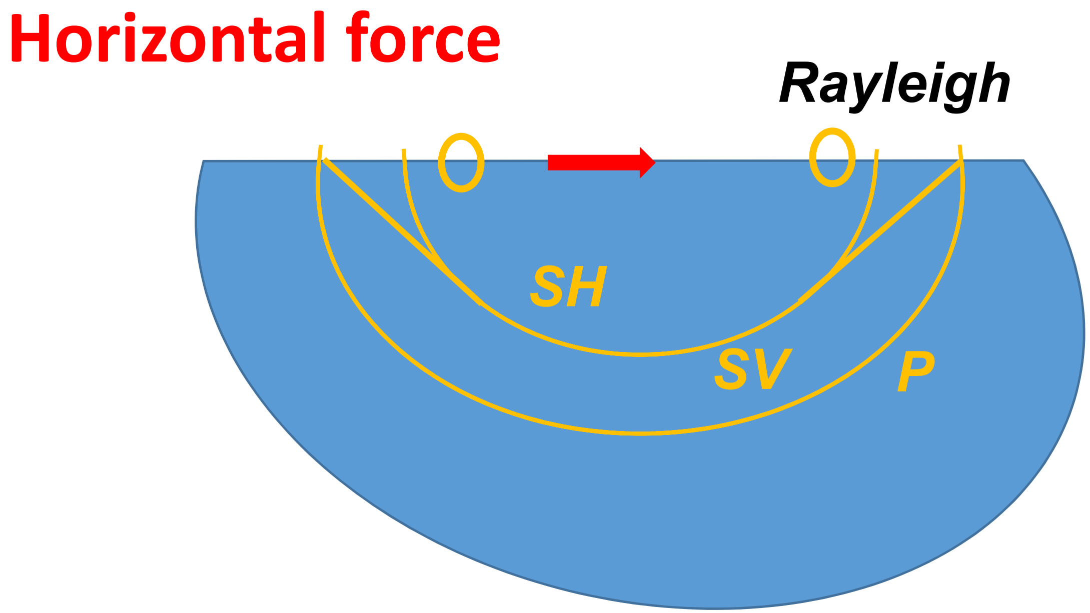
Lamb(1904)

Miller & Pursey (1955)

Weaver (1985)

SH=0 %   R=67%   SV=26% P=7%

Deterministic Partition of Energy



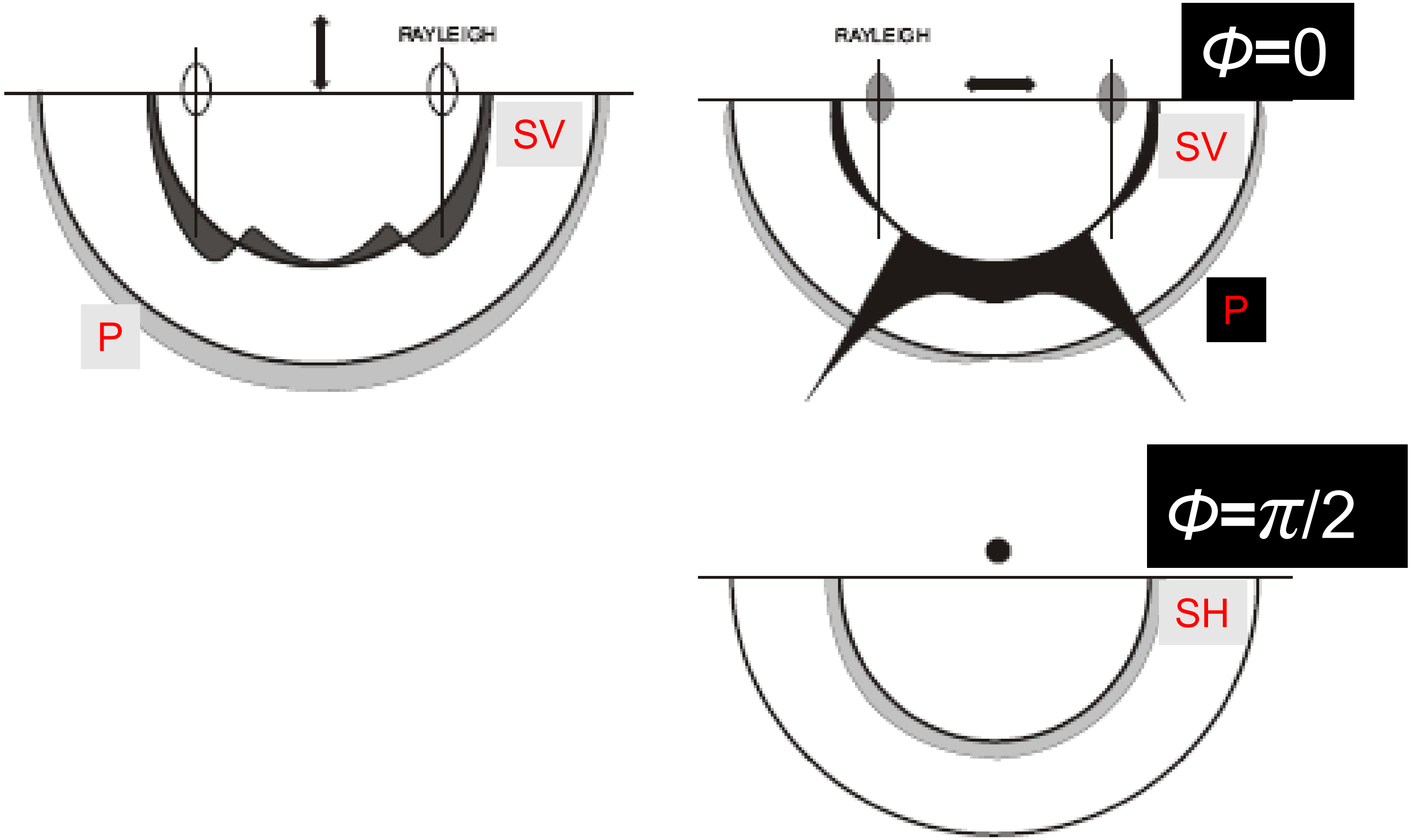
Chao(1960)

Cherry (1962)

Weaver (1985)

SH=60% R=18% SV=16% P=6%

Sánchez-Sesma et al (2011) BSSA



Sánchez-Sesma *et al.* (2011) BSSA

Seismic Noise and H/V

A Theory for H/V

With **Directional Energy Densities** one can compute the H/V ratio as:

$$[H/V](\mathbf{x};\omega) = \sqrt{\frac{E_1(\mathbf{x};\omega) + E_2(\mathbf{x};\omega)}{E_3(\mathbf{x};\omega)}}$$

measurements ←→ system properties

$$[H/V](\mathbf{x};\omega) = \sqrt{\frac{\mathrm{Im}[G_{11}(\mathbf{x},\mathbf{x};\omega)] + \mathrm{Im}[G_{22}(\mathbf{x},\mathbf{x};\omega)]}{\mathrm{Im}[G_{33}(\mathbf{x},\mathbf{x};\omega)]}}$$

Sánchez-Sesma et al. (2011)
3D problem (BW & SW)

Kawase et al. (2011)
1D problem (BW)

Computation of ImG$_{11}$, and ImG$_{33}$ by an integral on the radial wavenumber

$$\mathrm{Im}\left[G_{11}(r,0,0;0;\omega)\right] = \mathrm{Im}\left[\underbrace{\frac{\mathrm{i}}{4\pi}\int_0^{+\infty} f_{SH}(k)\left[J_0(kr)+J_2(kr)\right]dk}_{SH} + \underbrace{\frac{\mathrm{i}}{4\pi}\int_0^{+\infty} f_{PSV}^{H}(k)\left[J_0(kr)-J_2(kr)\right]dk}_{PSV}\right]$$

$$\mathrm{Im}\left[G_{33}(r,0,0;0;\omega)\right] = \mathrm{Im}\left[\frac{\mathrm{i}}{2\pi}\int_0^{+\infty} f_{PSV}^{V}(k)J_0(kr)dk\right]$$

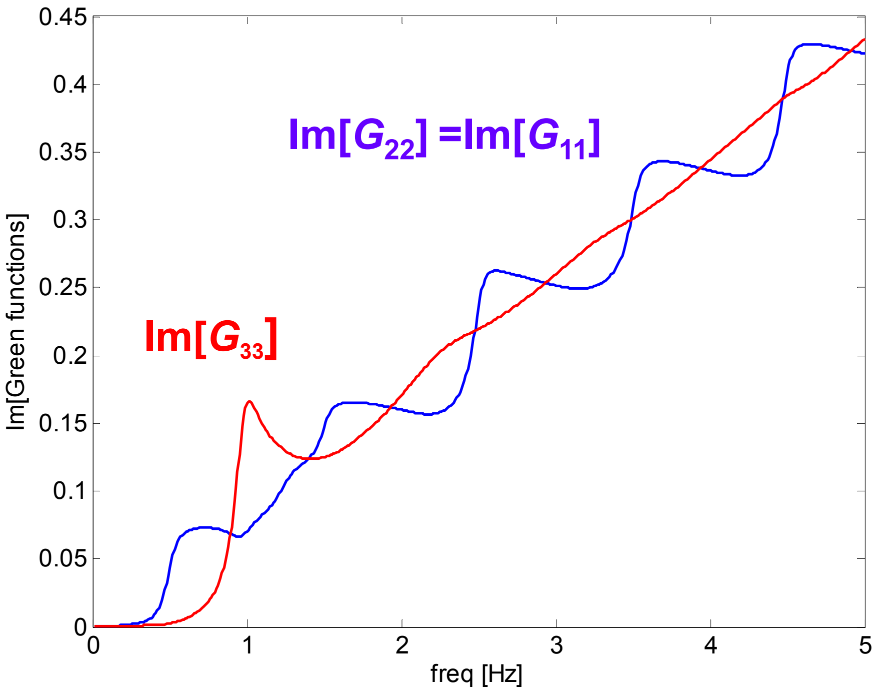
$$f_{PSV}^{V}(k) = -\frac{[GN-LH]}{[NK-LM]},\quad f_{PSV}^{H}(k) = \frac{[RM-SK]}{[NK-LM]},\quad f_{SH}(k) = \frac{(J_L)_{12}-(J_L)_{22}}{(J_L)_{21}-(J_L)_{11}}$$

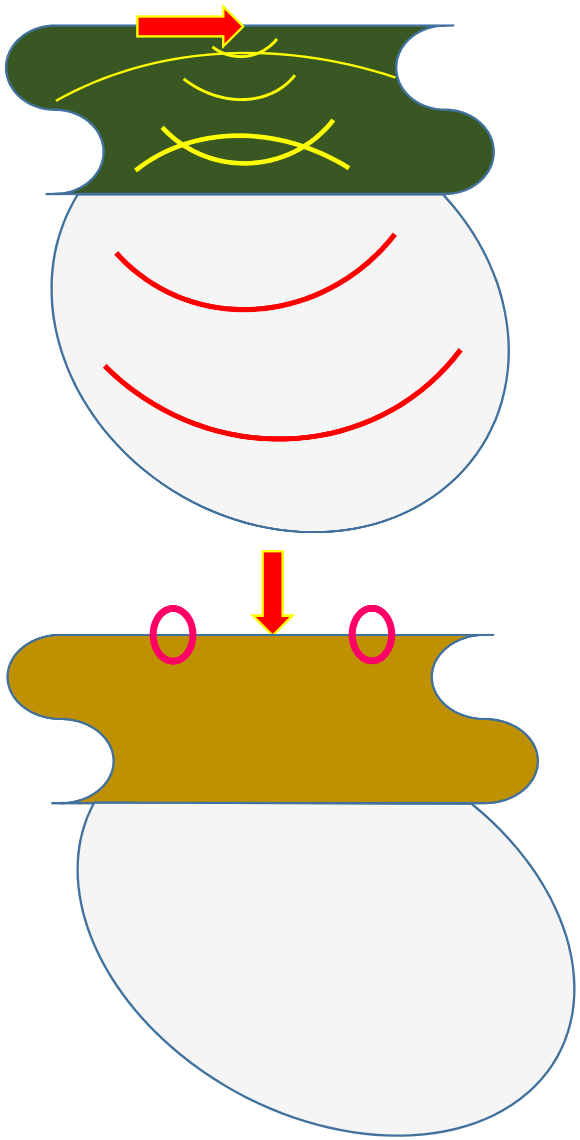
Harkrider (1964)

Layer over Half-space

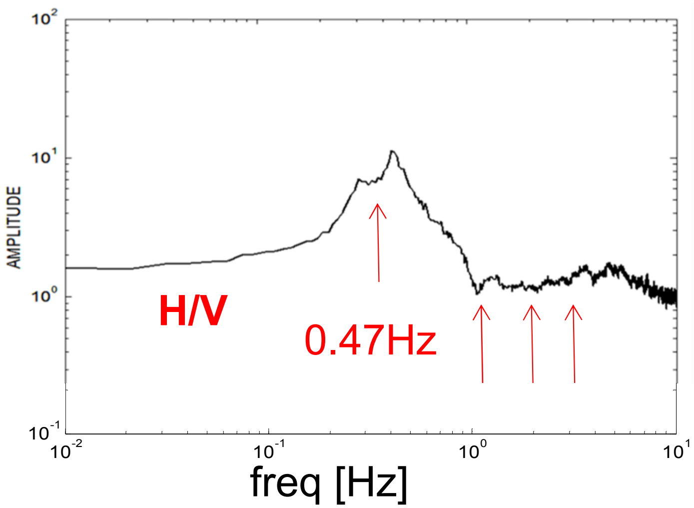
$\mathrm{Im}[G_{11}(0,0,\omega)]$, $\mathrm{Im}[G_{33}(0,0,\omega)]$

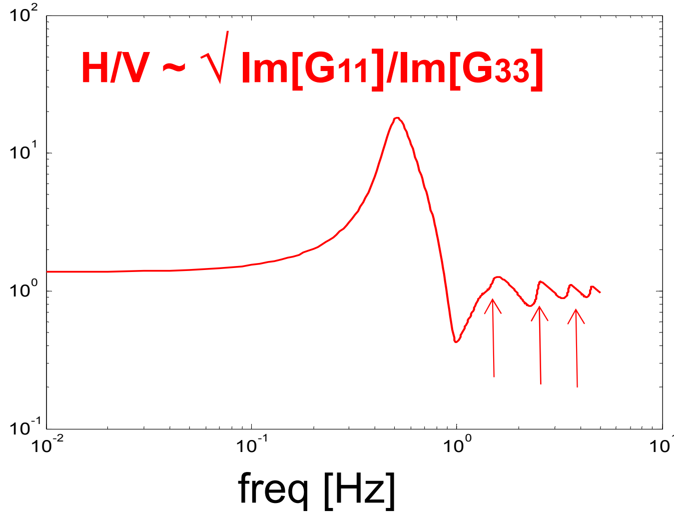
3D Solution

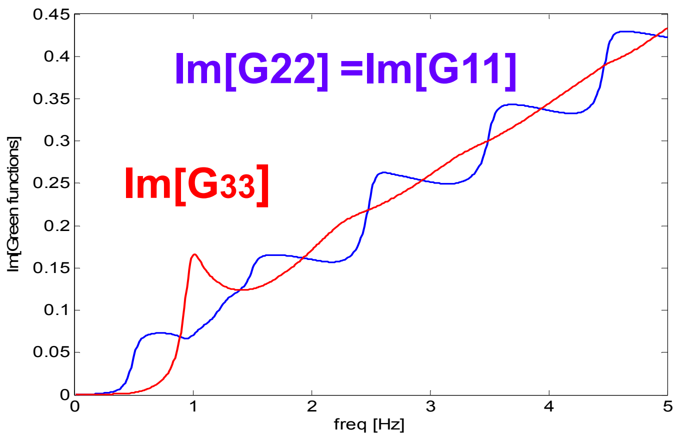




The Texcoco Experiment







Sánchez-Sesma et al. (2011)

3D Effect !

Layer over half-space

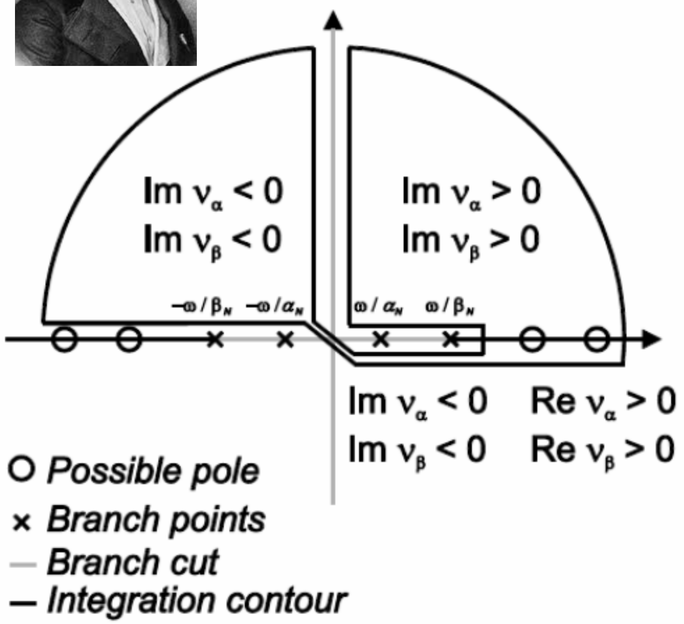
$$\nu_{\beta_N} = \sqrt{k^2 - (\omega / \beta_N)^2}$$

$$\nu_{\alpha_N} = \sqrt{k^2 - (\omega / \alpha_N)^2}$$

$$\mathrm{Im}[G_{11}^{PSV}(0;0;\omega)] = \mathrm{Im}[G_{22}^{PSV}(0;0;\omega)] = -\frac{1}{4}\sum_{m\in \mathrm{RAYLEIGH}} A_{Rm}\chi_m^2 + \frac{1}{4\pi}\int_0^{\omega/\beta_N} \mathrm{Re}\left[f_{PSV}^{H}(k)\right]_{4^{th}qu} dk$$

$$\mathrm{Im}[G_{11}^{SH}(0;0;\omega)] = \mathrm{Im}[G_{22}^{SH}(0;0;\omega)] = -\frac{1}{4}\sum_{m\in \mathrm{LOVE}} A_{Lm} + \frac{1}{4\pi}\int_0^{\omega/\beta_N} \mathrm{Re}\left[f_{SH}(k)\right]_{4^{th}qu} dk$$

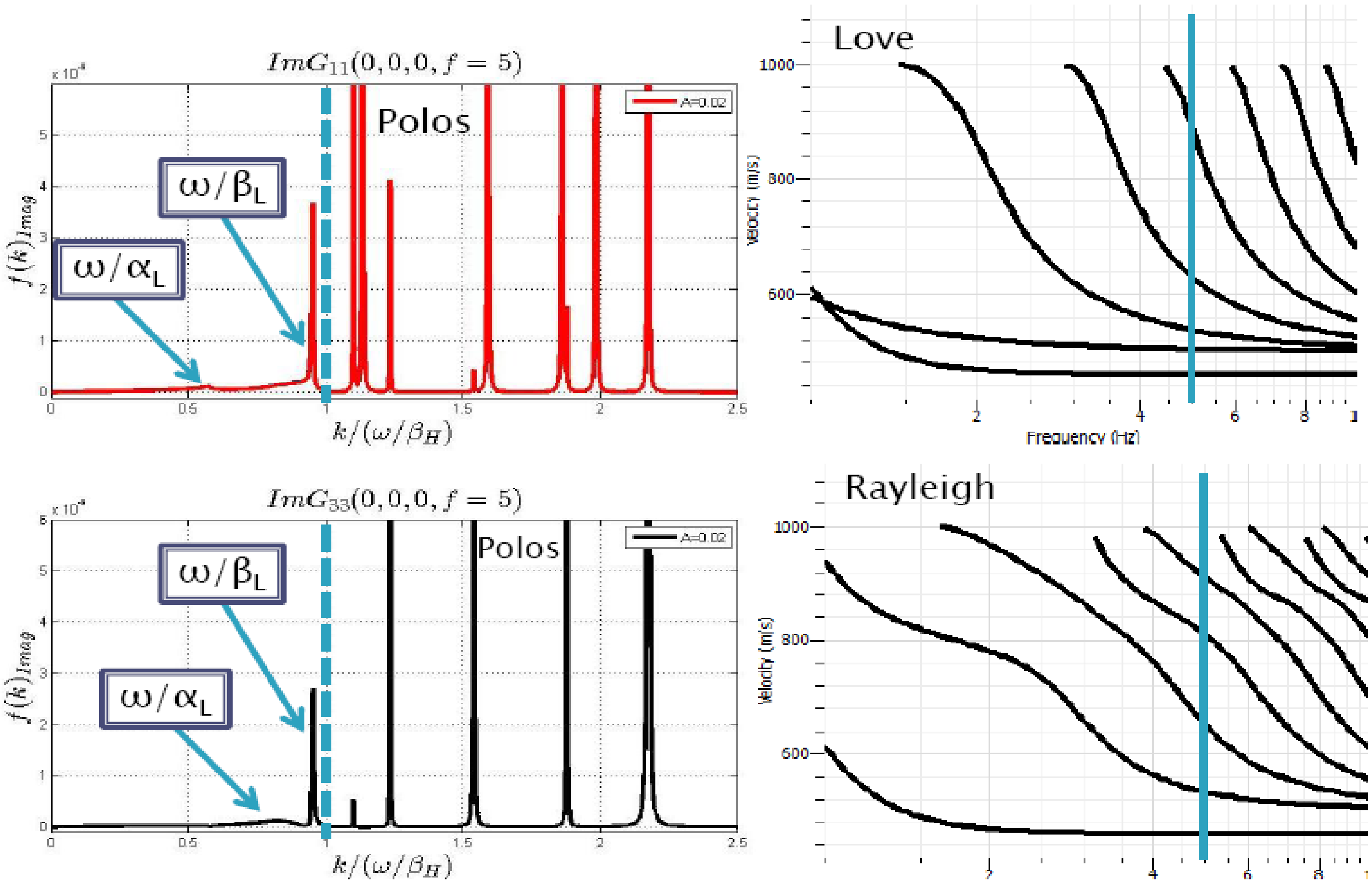
$$\mathrm{Im}[G_{33}(0;0;\omega)] = -\frac{1}{2}\sum_{m\in \mathrm{RAYLEIGH}} A_{\mathrm{Rm}} + \frac{1}{2\pi}\int_0^{\omega/\beta_N} \mathrm{Re}\left[f_{PSV}^{V}(k)\right]_{4^{th}qu} dk,$$

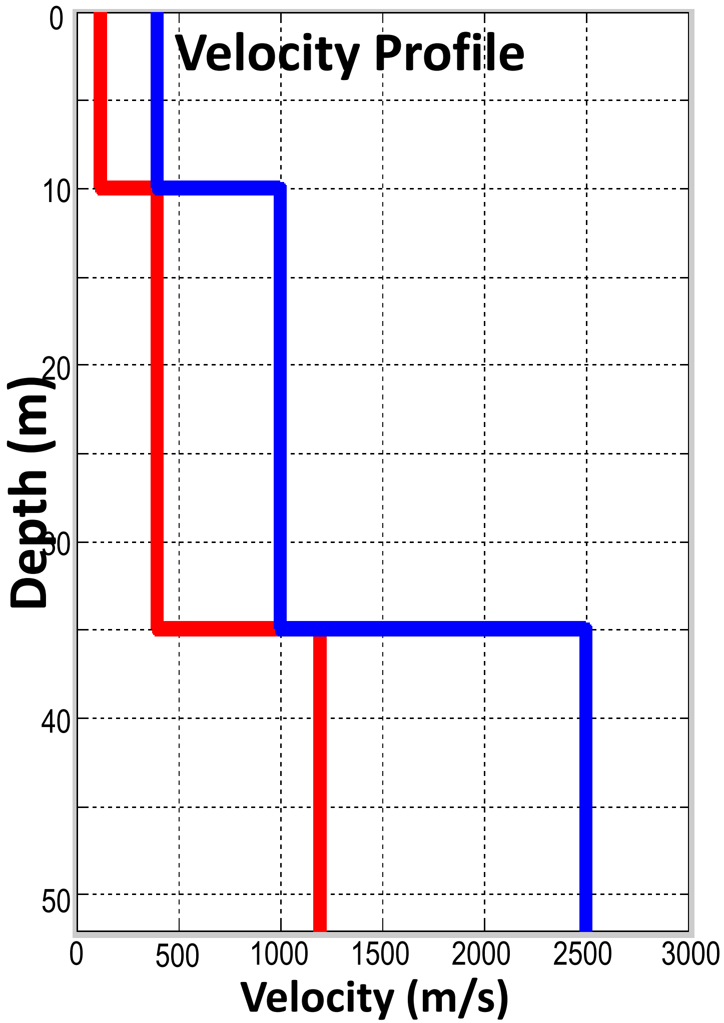


Fast computation of $\mathrm{Im}[G_{ij}(0,0,\omega)]$
with Cauchy Residue Theorem
An oportunity to speed up inversion

García-Jerez et al (2013)

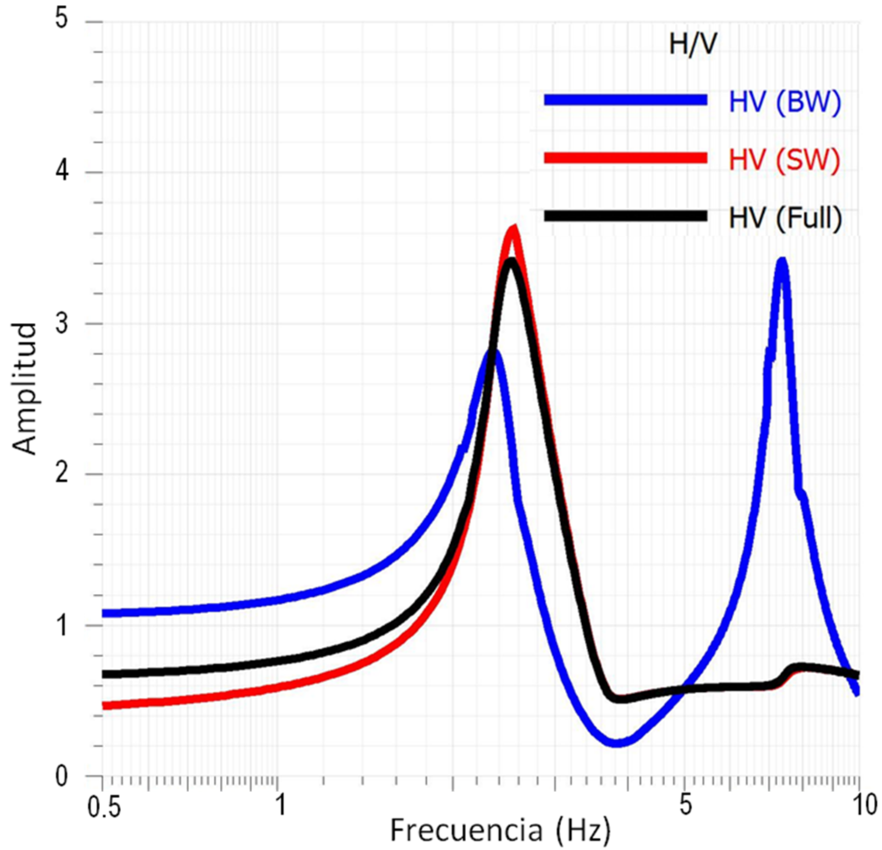
Poles localization – Dispersion Curves

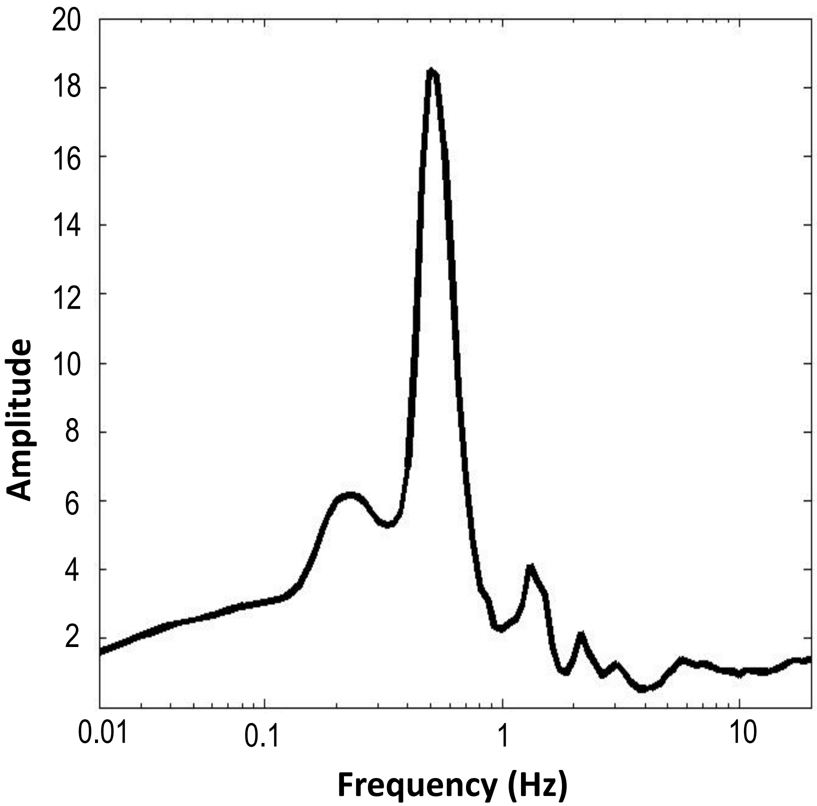




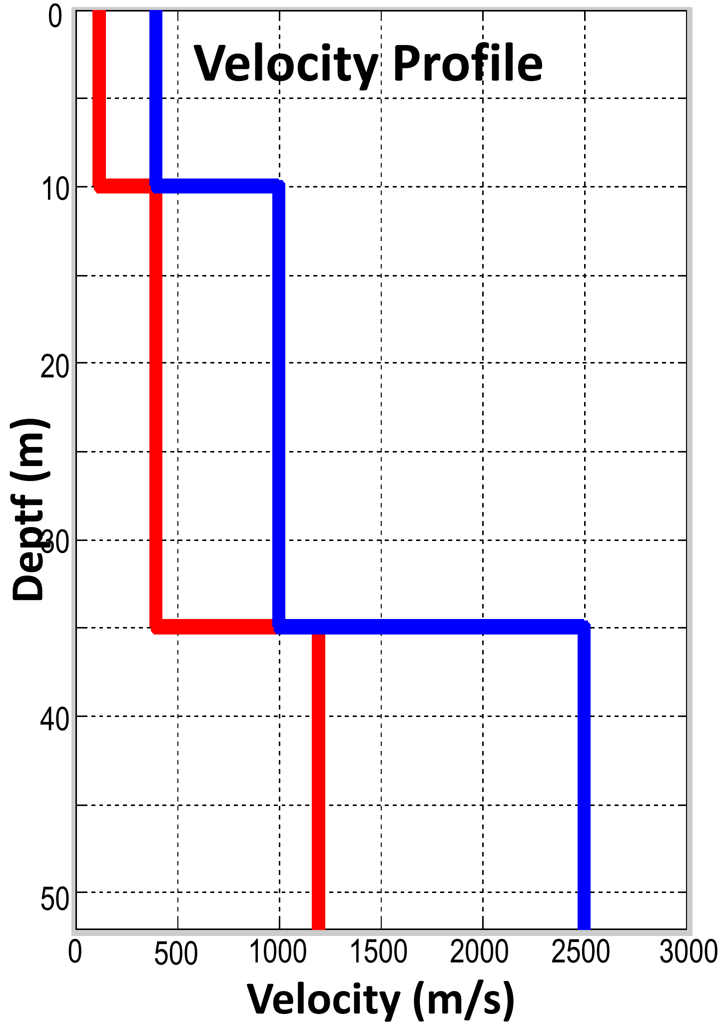
Direct Problem

$$\frac{H}{V} = \frac{H_{body} + H_{Love} + H_{Rayleigh}}{V_{body} + V_{Rayleigh}}$$

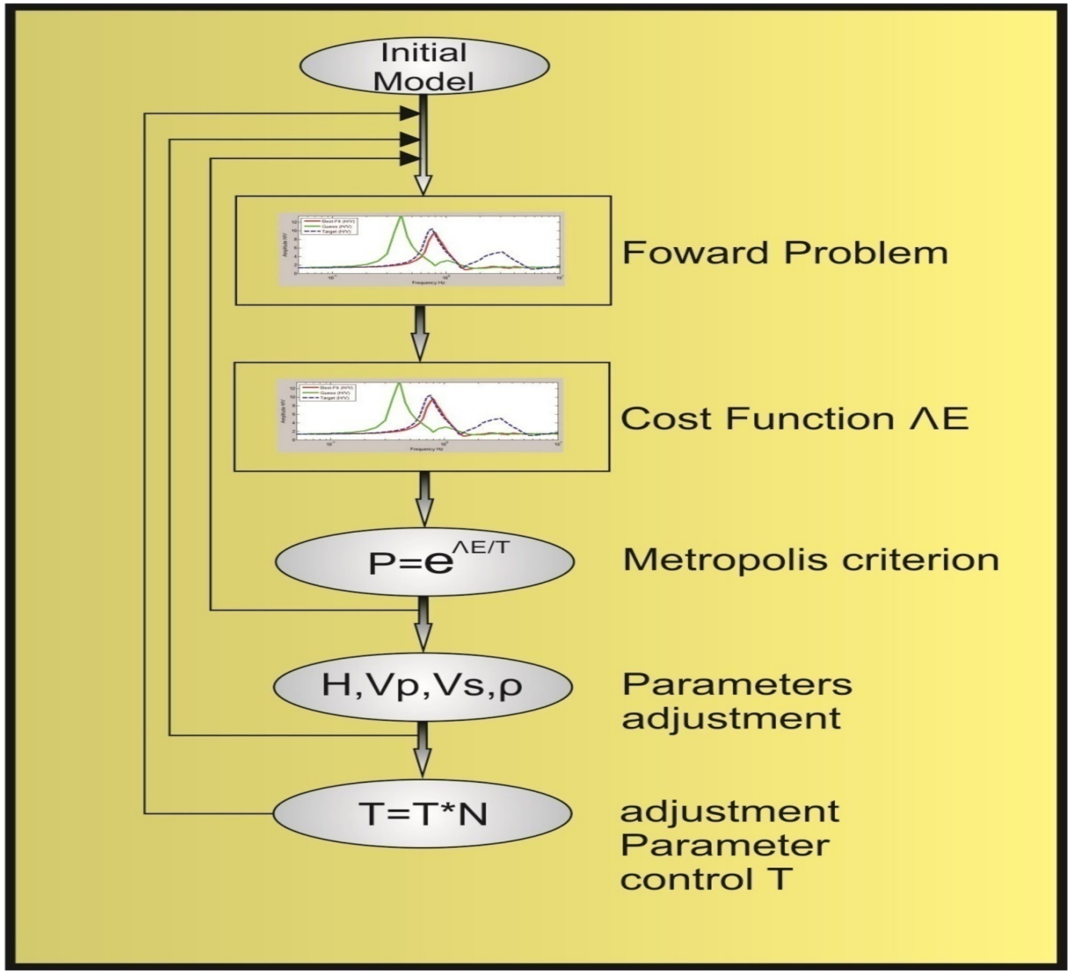




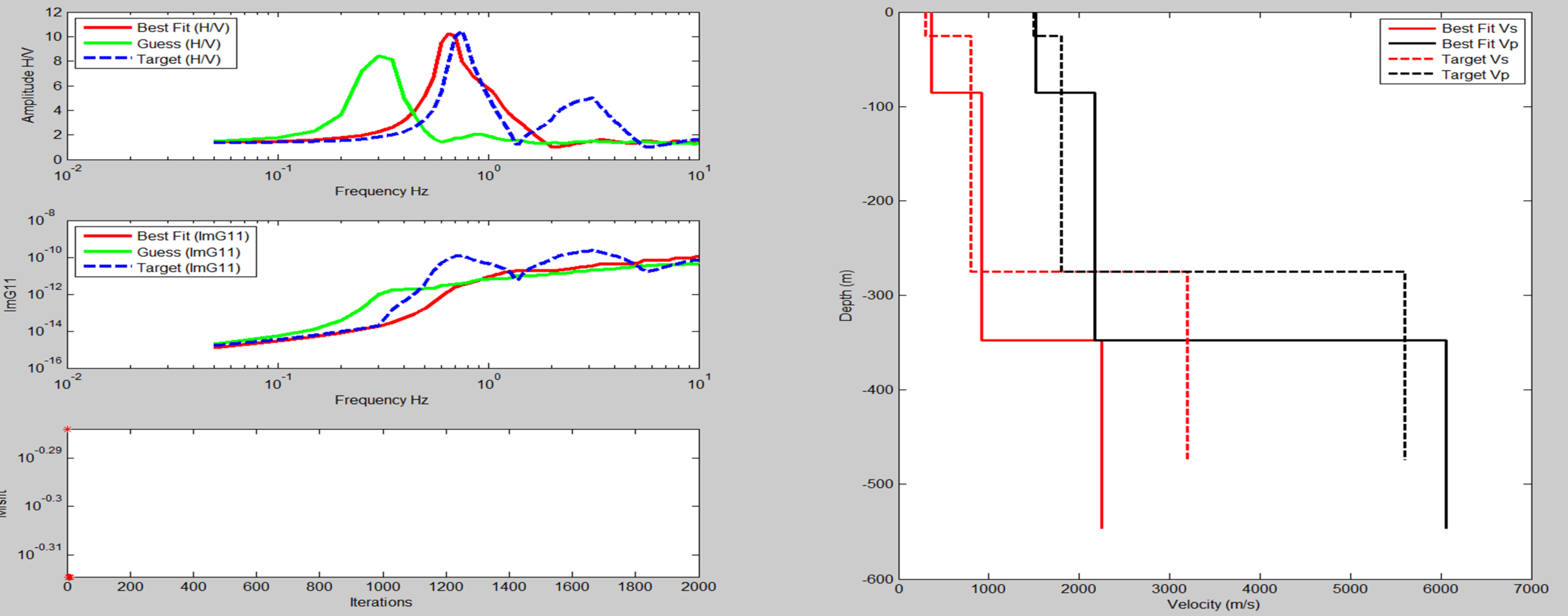
Inverse Problem



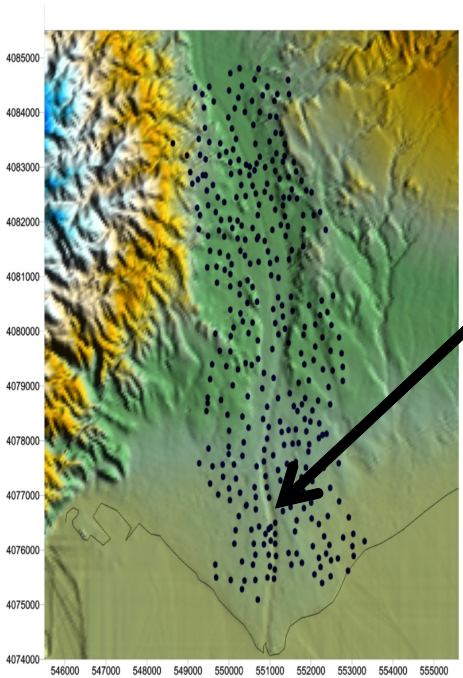
Global optimization using simulated annealing



Inversion using Simulated Annealing (Piña et al., 2015)

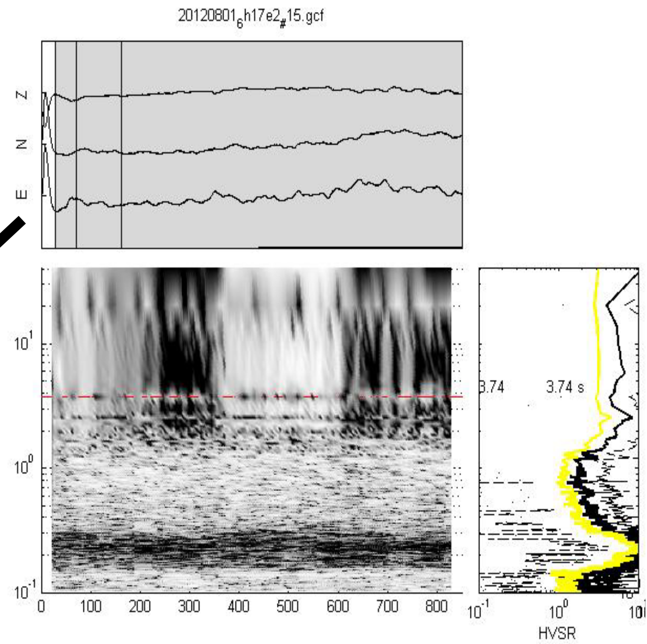


Application to site effect characterization at Almería, Andarax River, Spain



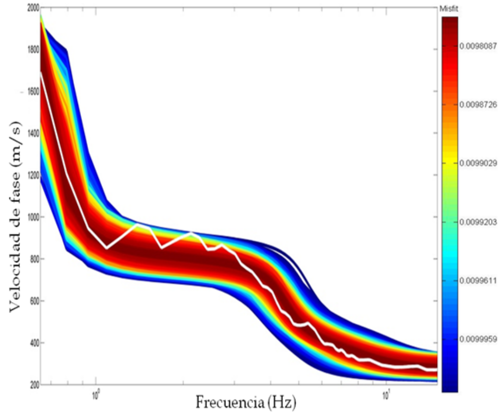
Stations distribution

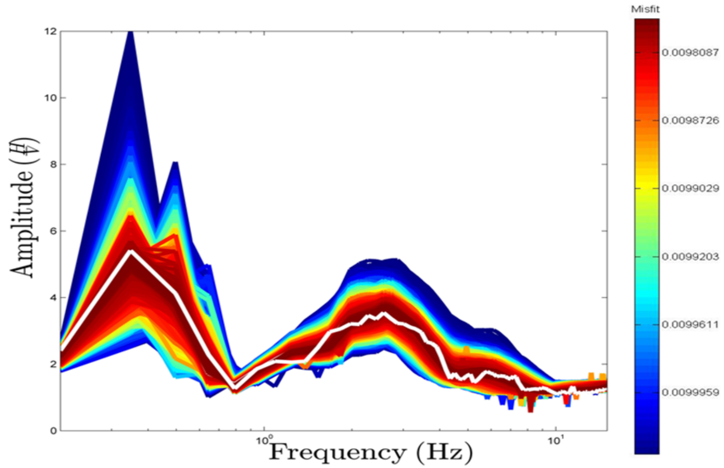
Stability of H/V



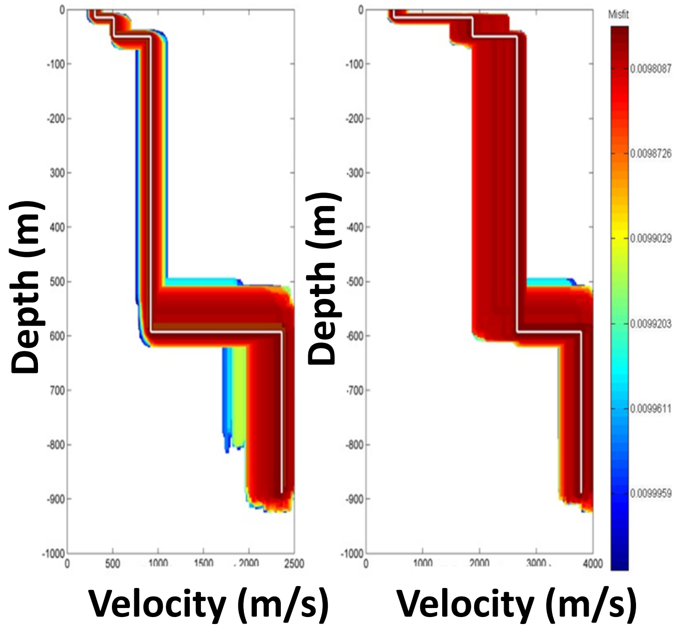
FTAN H/V
(from Dr. E. Carmona)

Application to site effect characterization at Almería, Andarax River, Spain



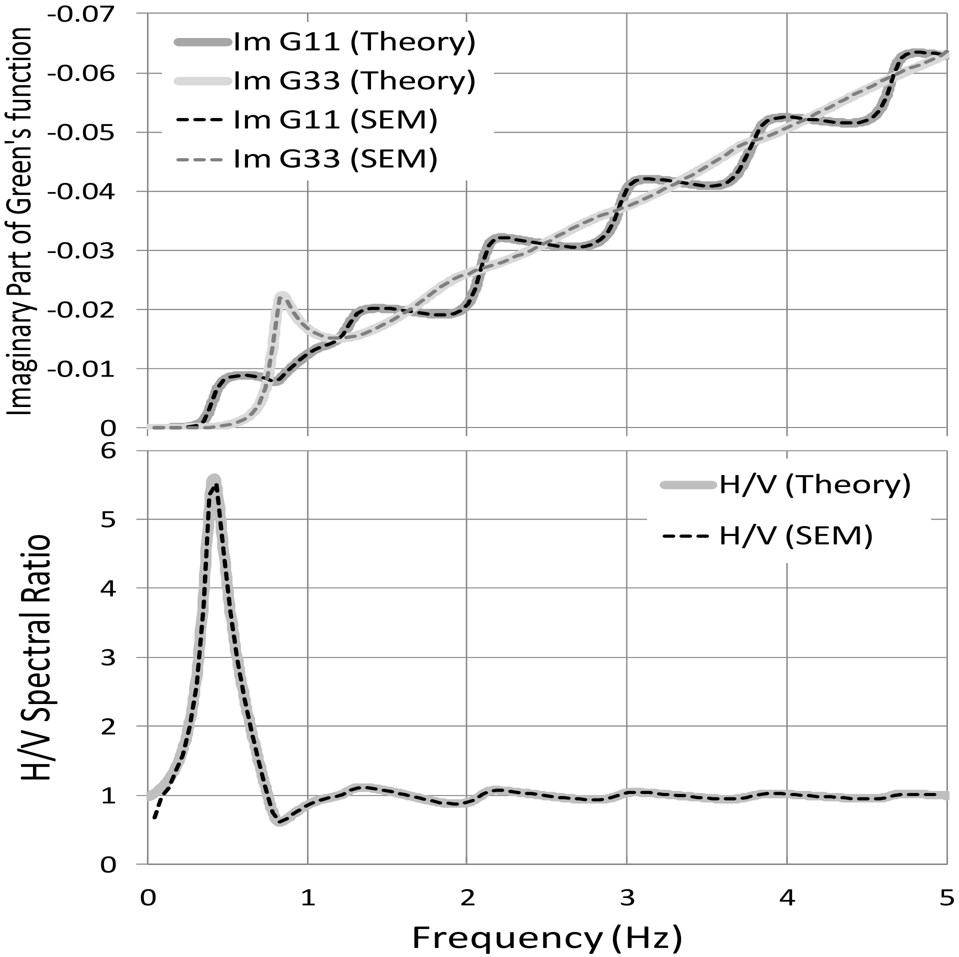


SPAC - 5 vertex, Rmax 450m



H/V with lateral irregularity

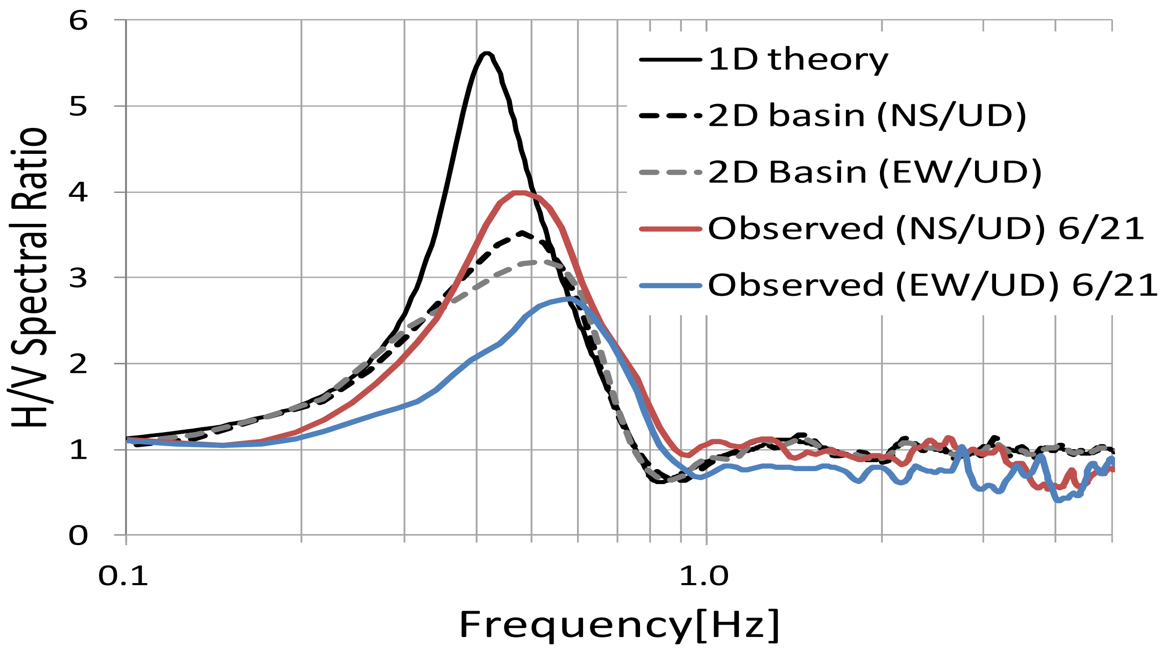
Verification of 1D results with numerical modeling



- **Imaginary Parts of Green's function**
  - **Good agreement between theory and SEM for G11 y G33**
- **Spectral Ratio H/V**
  - **Excellent agreement between theory and SEM**

Matsushima et al. (2014)

H/V with lateral irregularity



- Spectral ratios H/V
  - The efects of lateral irregularity are clear in NS/UD and EW/UD
  - Peak Amplitudes
    - NS/UD > EW/UD
  - Peak Frequency
    - NS/UD < EW/UD
  - Qualitative Agreement

Matsushima et al. (2014)

Comments and Conclusions (1)

The Principle of Equipartition of Energy allows characterization of diffuse fields. In seismology a key issue is Multiple Scattering.

We examined the Properties of the Equipartition in a full-space, a half-space and we discussed the experimental verification.

We reviewed retrieval of the Green's function from the average of correlations in a diffuse field (coda, noise) or for earthquakes with dominance of body waves. Generalized diffuse field.

Deterministic Gij with equipartitioned plane wave cocktails.

Comments and Conclusions (2)

Directional Energy Densities from autocorrelation averages.

- **Deterministic partition of Energy**
- **H/V ratios**
  - **For seismic noise → fast calculation of H/V → inversion**
  - **For incoming body waves → fast calculation → inversion**
  - **Effects against depth → fast calculation → inversion**
  - **Effects in the presence of lateral irregularities**

Without a doubt, these concepts will keep surprising us with new applications.

Thank you ☺..!