AV-07

OPTIMAL SENSOR PLACEMENT FOR BUILDING STRUCTURAL DAMAGE ASSESSMENT

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INTRODUCTION

Recent research shows that for the purpose of structural damage assessment, modal tests are needed to gain the dynamic properties of the structures. Due to economic constraints, it is often required to select a predefined number of sensor locations for damage monitoring.

Based on linear model estimation, this paper chooses the target mode shapes as a design matrix of the linear model. The singular value decomposition method is used to decompose the design matrix, and the improved reduced system is used to significantly reduce the degrees of freedom of the system while maintaining calculation accuracy. A 15-story 2-bay steel frame structure is used as an example to illustrate this proposed hybrid algorithm for building structures.

LINEAR MODEL ESTIMATION

1. Linear Gauss Model and Fisher Information Matrix

Suppose the relationship between an unknown vector β and the measured vector Y can be expressed in the formulation of the linear Gauss model:

$$Y = X \cdot \beta + e \tag{1}$$

where, X is the design matrix. $\beta \sim N(E\beta, P_\beta)$, $e \sim N(0,R)$ is the zero-mean mearsurement noise. The error covariance of estimator of β is:

$$P_{\tilde{\beta}} = (X^T R^{-1} X)^{-1} \tag{2}$$

If $\hat{\beta}$ is a random unbiased estimator of determinate variation β based on measurement vector Y, the covariance of estimator error $\tilde{\beta} = \beta - \hat{\beta}$ has a lower limit, that is:

$$P_{\tilde{\beta}} \geq M_\theta^{-1} \tag{3}$$

The inequality (3) is known as the Cramer-Rao bound. The matrix M_θ is the Fisher information matrix. Estimator of $\hat{\beta}$ is effective only if $\hat{\beta}$ satisfies the equal condition of the Cramer-Rao bound. For the linear Gauss model, it had been verified that Fisher information matrix is the inverse of error covariance matrix.

From those analysis, it can be found that if $\hat{\beta}$ is the effective unbiased estimator of β, the error covariance will be the least, thus, it is the best estimator of β.

2.Singular Value Decomposition (SVD) of design matrix X

The design matrix X can be decomposed using SVD:

$$X = H \cdot \Lambda^{\frac{1}{2}} \cdot G^T \tag{4}$$

Substitute (4) into the original linear model, $\Lambda^{\frac{1}{2}} \cdot G^T \cdot \beta = \alpha$, a new linear model can be gained:

$$Y = H \cdot \alpha + \mathrm{e} \tag{5}$$

in which, the new design matrix H is of a column-full rank.

$$R^2 = \frac{\mathrm{SSR}}{\mathrm{SST}} = \frac{Y^\mathrm{T} \cdot H \cdot H^\mathrm{T} \cdot Y}{Y^\mathrm{T} \cdot Y} = \sum_{i=1}^{n} Y^\mathrm{T} \cdot h_i \cdot h_i^\mathrm{T} \cdot Y \Big/ \|Y\|^2 \tag{6}$$

where, h_i is the ith row of H.

The total correlation coefficient R^2 is decomposed into the contribution of each row of H. Each component represents the contribution of the ith row of H to the total R^2. The row with the smallest diagonal value of $H \cdot H^T$ is selected for the first elimination. The appropriate row in H, namely appropriate DOF in the target modes, is then removed. The process is repeated until the remaining numbers of DOFs reach the final set of desired numbers of DOFs.

HYBRID ALGORITHM

1. Improved Reduced System (IRS)

The reduction matrix in the Improved Reduced System is

$$D_\mathrm{I} = D_\mathrm{G} + K_\mathrm{ss}^{-1}\left(M_\mathrm{sm} + M_\mathrm{ss} D_\mathrm{G}\right) \cdot M_\mathrm{G}^{-1} K_\mathrm{G} \tag{7}$$

In this method, the slave coordinates can be expressed as a function of the master coordinates:

$$\{x_\mathrm{s}\} = \left[D_\mathrm{G} + K_\mathrm{ss}^{-1}\left(M_\mathrm{sm} + M_\mathrm{ss} D_\mathrm{G}\right) \cdot M_\mathrm{G}^{-1} K_\mathrm{G}\right] X_\mathrm{m} = D_\mathrm{I} \cdot \{x_\mathrm{m}\} \tag{8}$$

From (7), we find that the reduction matrix in IRS includes not only stiffness information but also mass information of the original system. Thus, IRS translates both strain energy and kinetic energy of the original model to the reduced model.

2. Hybrid algorithm

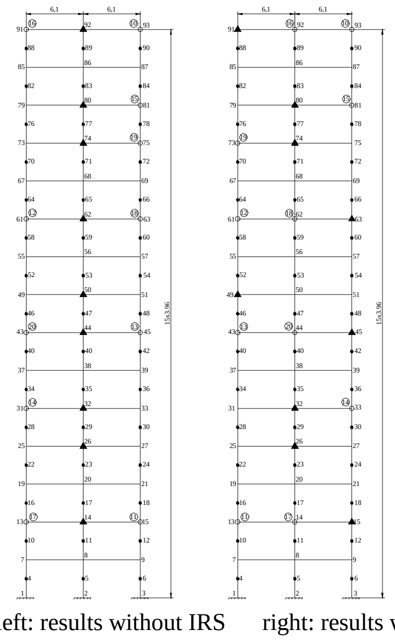
Solving the eigenproblem of the reduced model, the mode frequencies and mode shapes can be obtained. The mode shapes corresponding to different frequencies are of linear independence. Then, the singular value decomposition method is used to calculate the contribution of each sensor location and the optimal sensor locations can be obtained through iterative calculation.

EXAMPLE ILLUSTRATION

A 15-story, 2-bay steel frame structure with each span of 6.10m and floor height of 3.96m is illustrated. Figure 1 shows the calculation results of unreduced model and reduced model. Taking Fig. 1 left as an example, first we put the first 6 basic sensors on the building, that is, at nodes 14, 32, 44, 62, 74, and 92. If other additional sensors are available, the sequence of choosing the locations is at nodes 26, 80, 50, 93, and so on.

CONCLUSIONS

The hybrid algorithm of sensor locations method using both IRS and SVD significantly reduces the DOFs of the model and shows higher numerical stability. It is applicable for building structures.



(left: results without IRS right: results with IRS)

Figure 1 Sensor placements sequence

(numbers without circles are series of finite element modeling, ▲s are the first important 9 sensor locations, the numbers with circles are 10th~20th sensors chosen in sequence.)