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Statistical Research on S-Wave Incident Angle

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Synopsis

In this paper, the following assumptions are made: 1. seismic source is a point one; 2. the near-field seismic waves are body ones; and 3. the medium of the earth is a half-space which is homogeneous and elastic. According to the above-mentioned assumptions, this paper obtain a theoretical formula to estimate S-wave incident angle by using the record of strong ground motion. During the process of calculations, the paper also presents a method of getting in-plane and anti-plane from the record of strong ground motion. By making use of 214 records of strong ground motion in the western American, this paper gives a statistical relation among S-wave incident angle, epicentral distance and frequency. The results show that the average value of S-wave incident angle is about 57°, and has little to do with the epicentral distance.

Key Words: incident angle, S-wave, ground motion, epicentral distance, frequency

1. Introduction

In order to estimate the effects of local site conditions on ground motion and analyze the interaction of soil and structure in aseismic design, it is transcendentally assumed in the current engineering that shear wave incidences vertical upwards. Estimation of seismic wave incident angles using observed data is an open question in the research of engineering seismology. In addition, as far as the design of seismic motion field in large scale engineering site is concerned, e.g., in the processes of synthesizing ground rotation time history from ground translation history, it is necessary to estimate incident angle of seismic wave (Jin and Liao, 1991). This paper will do a basic discussion of the above question. The seismic wave input into engineering site usually consist of body wave and surface wave. As far as near-field seismic motion is concerned, the paper considers only the contribution of body wave as approximate of the first class.

According to Wale integral (Aki and Richards, 1980), spherical wave radiated from a seismic source can be regarded as the superposition of a series of both homogeneous and inhomogeneous plane waves. Seismic motions actually recorded at the stations indicate the extreme irregularity of ground motion time history. However, in terms of vibration decomposition, the time can be viewed as the superposition of a series of plane harmonic waves. This paper will discuss, in the light of the elastic wave theory, the link between the ground motion and the incident angle of these harmonic waves, and the statistical principle of S-wave incident angle and epicentral distance and frequency change on the basis of ground motion records.

2. Theoretical Equations of the Estimation of S-Wave Incident Angle on the Basis of Ground Motion Records

It is easy to obtain the structure of average speed from the seismic source to a station based on the strong seismic observations (Jin and Liao, 1990). Thus the terrestrial medium can be simplified as harmonicisotropic half-space. As for the in-plane problem, it is assumed that plane P-wave and S-wave

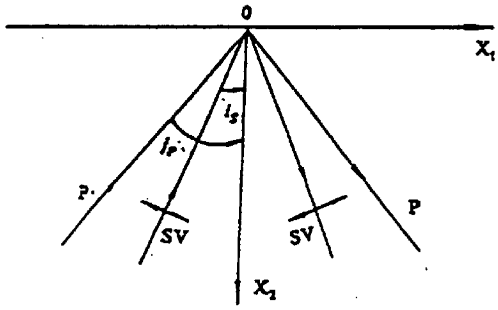

Fig. 1 P-, SV-wave system incident upon free ground surface

travel along X_1 in an elastic half-space, as shown in Fig. 1, and its scalar potential function and vector potential function are respectively shown as follows:

$$\varphi\left(X_1,X_2,\omega,t\right)=\left(A_p e^{iK_pX_2}+B_p e^{-iK_pX_2}\right)e^{i\omega(t-X_1/c)}$$
$$\Psi\left(X_1,X_2,\omega,t\right)=\left(A_s e^{iK_1X_1}+B_s e^{-iK_1X_2}\right)e^{i\omega(t-X_1/c)} \tag{1}$$

where $K_p=\frac{\omega}{c}\left[\frac{c^2}{\alpha^2}-1\right]^2$, $K_s=\frac{\omega}{c}\left[\frac{c^2}{\beta^2}-1\right]^{1/2}$, c is the phase velocity traveled along X_1 axis. β and α are the S-wave velocity and P-wave velocity, respectively. A_p and B_p represent the incident and reflected wave amplitude of P-wave. A_s and B_s represent the incident and reflected wave amplitude of S-wave. For the sake of convenience, the potential function of the incident upon the free ground surface of both S-wave and P-wave are unified as Eq. (1), although actually P-wave and S-wave travel independently. Let $A_s=0$, then it is equivalent to the case when there is only P-wave incident. Let $A_p=0$, then it is equivalent to the case when there is only S-wave incident. With the relation between displacement and potential function, as well as zero stress of the free ground surface, the displacement of the ground surface indicated by incident wave amplitude parameters A_p and A_s is as below:

$$U_1=\frac{2i}{a-b}\left(K_s-\frac{a\omega}{c}\right)\left(A_p+bA_s\right)e^{i\omega(t-X_1/c)}$$
$$U_2=\frac{2i}{a-b}\left(bK_p-\frac{\omega}{c}\right)\left(A_p-aA_s\right)e^{i\omega(t-X_1/c)} \tag{2}$$

in which $a=\frac{\gamma_2K_s}{\gamma_1}$, $b=\frac{\gamma_1}{\gamma_2K_p}$, $\gamma_1=\omega_2\left[\frac{2\beta^2}{c^2}-1\right]$, $\gamma_2=2\beta^2\frac{\omega}{c}$. Now, let us consider the situation when P-wave and SV-wave make an individual incidence.

P-wave incidence: Let $A_s=0$ in Eq. (1), and assume P-wave incidence angle to be j_p, the reflected angle of reflected SV-wave to be j_s, then, $c=\frac{\beta}{\sin j_s}=\frac{\alpha}{\sin j_p}$. If $\lambda_p=\frac{U_2^p}{U_1^p}$ is determined, with U_2^p and U_1^p standing respectively for the displacement along axis X_2 and axis X_1 created by P-wave incidence individually, then from Eq. (2) we can have:

$$\lambda_p=-ctg2j_s \tag{3}$$

SV-wave incidence: let $A_p=0$ in Eq. (1), and assume the incident angle of incident SV-wave to be i_s and the reflected angle of reflected P-wave to be i_p, then $c=\frac{\beta}{\sin i_s}=\frac{\alpha}{\sin i_p}$. If $\lambda_s=\frac{U_2^s}{U_1^s}$ is determined, with U_2^s and U_1^s standing respectively for the displacement along axis X_2 and axis X_1 created by SV-wave incidence individually, then similarly we have:

$$\lambda_s=\begin{cases}\dfrac{\sin i_s\left(\beta^2/\alpha^2-\sin^2 i_s\right)^{1/2}}{1/2-\sin^2 i_s} & 0<i_s<i_0\\[2ex] \dfrac{\sin i_s\left(\sin^2 i_s-\beta^2/\alpha^2\right)^{1/2}}{\sin^2 i_s-1/2} & i_0<i_s<\pi/2\end{cases} \tag{4}$$

where $i_0=\sin^{-1}(\beta/\alpha)$ is the limit angle of SV-wave incidence, $i=\sqrt{-1}$. When $i_s>i_0$, the reflected P-wave is changed into heterogeneous plane wave. In this case, $K_p=-i\frac{\omega}{c}\left[1-\frac{c^2}{\alpha^2}\right]^{1/2}$.

According to the superposition principle, when the incidence of P-wave and SV-wave reaches the ground surface, the total displacement of expressed as:

$$U_1=U_1^p+U_1^s$$
$$U_2=U_2^p+U_2^s \tag{5}$$

Definition: $W=U_2/U_1$. From Eqs. (4) and (5), we can have:

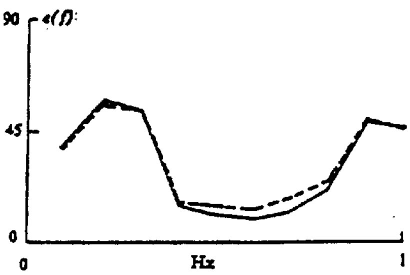

Fig. 2(a) The comparison between the true $e^0(f)$ curve (solid) and theoretical $e(f)$ curve (dotted)

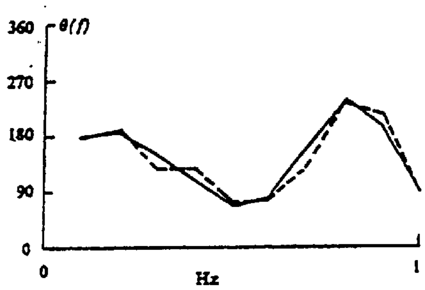

Fig. 2(b) The comparison between the true $\theta^0(f)$ curve (solid) and theoretical $\theta(f)$ curve (dotted)

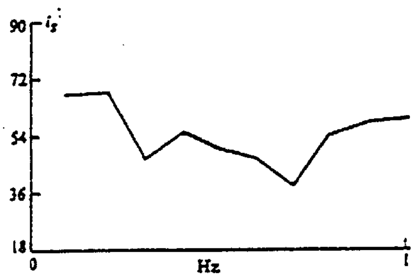

Fig. 2(c) The $i_s(f)$ curve obtained by calculation

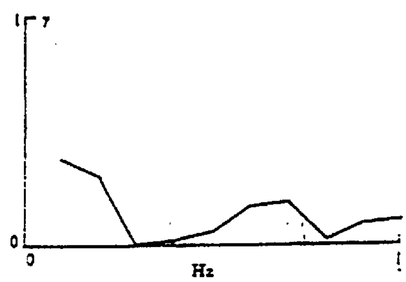

Fig. 2(d) The $\gamma(f)$ curve obtained by calculation

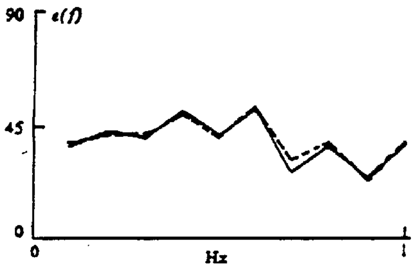

Fig. 3(a) The comparison between the true $e^0(f)$ curve (solid) and theoretical $e(f)$ curve (dotted)

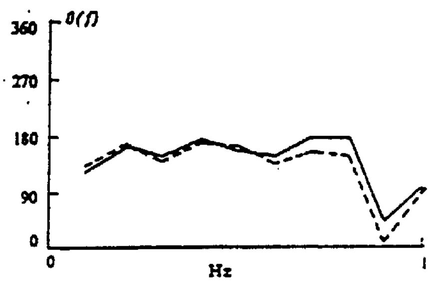

Fig. 3(b) The comparison between the true $\theta^0(f)$ curve (solid) and theoretical $\theta(f)$ curve (dotted)

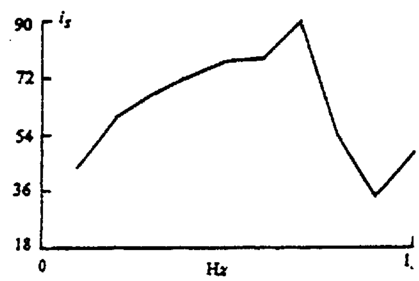

Fig. 3(c) The $i_s(f)$ curve obtained by calculation

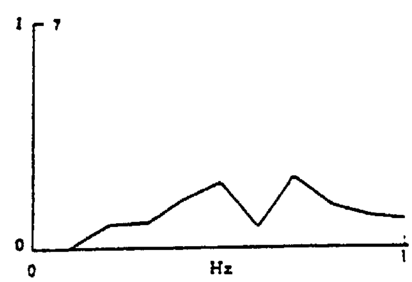

Fig. 3(d) The $\gamma(f)$ curve obtained by calculation

1. Let $e(i_s,\gamma) = tg^{-1}|W(i_s,\gamma)|$, $e^0 = tg^{-1}|W^0|$, then normalize $e(i_s,\gamma)$, e^0, $\theta(i_s,\gamma)$, and θ^0 (all units in degree), expressed as non-dimensional parameters, i.e.,

$$\begin{cases} f(i_s,\gamma) = e(i_s,\gamma)/90 \\ f^0 = e^0/90 \end{cases} \tag{14a}$$

$$W = \frac{\lambda_s + \lambda_p\left(U_1^p / U_1^s\right)}{1 + U_1^p / U_1^s} \tag{6}$$

In light of Eq. (2), it is easy to prove:

$$\frac{U_1^p}{U_1^s} = -\lambda_s y(i_s, j_s)\frac{A_p}{A_s} e^{i\omega\left(\frac{\sin j_p}{\alpha} - \frac{\sin j_s}{\beta}\right)X_1} \tag{7}$$

In Eq. (7):

$$y(i_s, j_s) = \begin{cases} \dfrac{tg2j_s\left(\beta^2/\alpha^2 - \sin^2 j_s\right)^{1/2}}{2tg2j_s\left(\beta^2/\alpha^2 - \sin^2 j_s\right)^{1/2} + \cos 2j_s} \cdot \dfrac{2tg2i_s\left(\beta^2/\alpha^2 - \sin^2 i_s\right)^{1/2} + \cos 2i_s}{tg2i_s\left(\beta^2/\alpha^2 - \sin^2 i_s\right)^{1/2}} & i_s < i_0 \\ \dfrac{tg2j_s\left(\beta^2/\alpha^2 - \sin^2 j_s\right)^{1/2}}{2tg2j_s\left(\beta^2/\alpha^2 - \sin^2 j_s\right)^{1/2} + \cos 2j_s} \cdot \dfrac{2tg2i_s\left(\sin^2 i_s - \beta^2/\alpha^2\right)^{1/2} + i\cos 2i_s}{tg2i_s\left(\sin^2 i_s - \beta^2/\alpha^2\right)^{1/2}} & i_s > i_0 \end{cases} \tag{8}$$

Let $X_1 = 0$, and substitute Eq. (7) into Eq. (6), we have:

$$W = \lambda_s \cdot \frac{1 - \lambda_p y(i_s, j_s) A_p / A_s}{1 - \lambda_s y(i_s, j_s) A_p / A_s} \tag{9}$$

In Eq. (9), A_p / A_s represents the ratio between P-wave incident wave amplitude and SV-wave incident wave amplitude, and it is usually a complex number. According to the earthquake source theory (Aki and Richards, 1980), if earthquake source is a point source dislocation, then $A_p / A_s = (\beta/\alpha)^3 \left(\Omega_p / \Omega_s\right) e^{i\omega R / V_\Psi}$, with Ω_p and Ω_s as directivity radiation factors of P-wave and S-wave respectively. Usually they vary with the change of the location of stations relative to the earthquake source. R is the seismic source distance, $V_\Psi = \frac{\alpha\beta}{\partial - \beta}$=. We can let $A_p / A_s = \gamma e^{i\theta_R}$, with $\theta_R = \omega R / V_\Psi$. Generally speaking, it is difficult to assess accurately the value of γ beforehand. As a direct judgment in physics, we assume that SV-wave and P-wave incidence reaches the ground surface with the identical angle. Thus, Eq. (9) can be written as

$$W = \lambda_s(i_s) \cdot \frac{1 - \lambda_p(i_s) y(i_s) \gamma e^{i\theta_R}}{1 - \lambda_s(i_s) y(i_s) \gamma e^{i\theta_R}} \tag{10}$$

where, $\lambda_p(i_s)$ and $y(i_s)$ as $j_s = \sin^{-1}(\beta / \alpha \sin i_3)$ from Eqs. (3) and (8) respectively. If $\beta/\alpha = 1/\sqrt{3}$ and β=3.2km/s. W will be the function of S-wave incident angle i_s, frequency ω, seismic source distance R and γ only. When we specified the seismic source distance R, i.e., a fixed station, and frequency $\omega = \omega_K (K = 0, 1, \cdots, M)$, theoretically, W is the function of i_s and γ only. Thus, Eq. (10) can be modified as:

$$W(i_s, \gamma) = \left|W(i_s, \gamma)\right| e^{i\theta(i_s, \gamma)} \tag{11}$$

In Eq. (11), $\left|W(i_s, \gamma)\right|$ and $\theta(i_s, \gamma)$ are amplitude spectrum and phase spectrum of $W(i_s, \gamma)$, respectively. If we can obtain the true seismic records $U_1^0(t)$ and $U_2^0(t)$ from the same station, their Fourier transformation will be $U_1^0(\omega)$ and $U_2^0(\omega)$, respectively. Therefore, with a given identical frequency $\omega = \omega_K$, we have:

$$W^0\Big|_{\omega = \omega_K} = \frac{U_2^0(\omega_k)}{U_1^0(\omega_k)} = \left|W^0\right| e^{i\theta^0} \tag{12}$$

In Eq. (12), superscript 0 indicates true seismic record. Being simultaneous with Eqs. (11) and (12), we have:

$$\begin{cases} \left|W(i_s, \gamma)\right| = \left|W^0\right| \\ \theta(i_s, \gamma) = \theta^0 \end{cases} \tag{13}$$

With the given frequency $\omega = \omega_K$, $\left|W^0\right|$ and θ^0 are two constants. Thus in principle, two parameters i_s and γ can be calculated by the solution of the nonlinear system of equations in Eq. (13). There are many ways to solve nonlinear system or equations, and one of the commonly applied solutions is iteration. But this solution is too sensitive to the selection of the initial value, and requires the specific expression of functional gradient. Therefore, the expression seems too complicated for the problem raised by this paper, and is not convenient for a large amount of statistical analysis. So, in our actual solution, we follow the following steps:

$$\begin{cases} g(i_s,\gamma) = \theta(i_s,\gamma) / 360 \\ g^0 = \theta^0 / 360 \end{cases} \tag{14b}$$

2. Define the objective function as

$$D(i_s,\gamma) = \left[f(i_s,\gamma) - f^0\right]^2 + \left[g(i_s,\gamma) - g^0\right]^2 \tag{15}$$

Set the step length Δi_s of i_s to be 1°, and the step length $\Delta\gamma$ of γ to be 0.05, let the computer search automatically within a two-dimensional plane of i_s, γ, with $0 \le i_s \le 90°$ and $0 \le \gamma \le 1.0$ so that the objective function $D(i_s,\gamma)$ reaches its minimum value, which corresponds to i_s, γ.

3. Since the observed value $W^0(\omega) = |W^0(\omega)| e^{i\theta^0(\omega)}$ varies with frequency, $i(\omega)$, $\gamma(\omega)$ can be obtained by repeating step 1 and step 2.

It is well known that local site effect has a greater influence on high-frequency seismic waves than on low-frequency seismic waves. In order to be consistent in the physical notations of the models selected in this paper, we filtered the selected seismic records in the paper, the frequency response function applied here is:

$$H(f) = \begin{cases} 0 & f < f_1 \\ 1 & f_1 \le f \le f_2 \\ 0 & f > f_2 \end{cases} \tag{16}$$

where f_1=0.1Hz, f_2=1.0Hz. Figures 2 and 3 are the changing curves of S-wave incident angle i_s and parameter γ in concordance with frequency in two seismic records. Thus, as long as $W^0(\omega) = \frac{U_2^0(\omega)}{U_1^0(\omega)} = |W^0(\omega)| e^{i\theta^0(\omega)}$ is known. S-wave incident angle $i_s(\omega)$ and $\gamma(\omega)$ can be determined by Eq. (13) and the above-mentioned method.

3. Method of Getting In-Plane and Anti-Plane Motions from the Record of Strong Ground Motion

The obtention of the vertical and horizontal components $a_1(t)$ and $a_2(t)$ of in-plane motion from true seismic records is a complex question and the key to inverse S-wave incident angle i_s as well. For the convenience of engineering application, an assumption of equivalent point source was made. In order to obtain a_2 and a_1 from the recorded

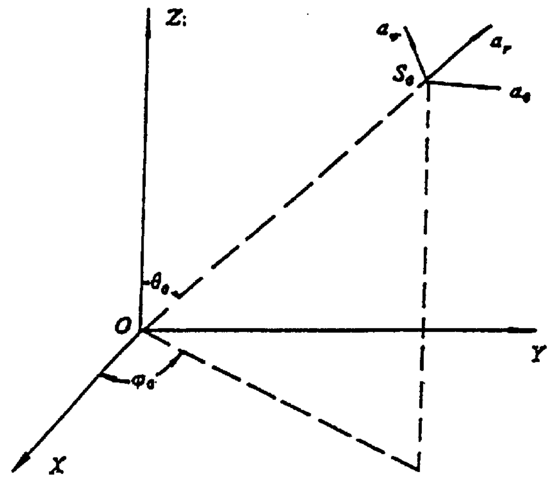

Fig. 4 Three component accelerations created by an effective point source at a site

component of south-north, east-west and vertical a_{NS}, a_{EW} and a_{UD}, the following coordinate transformation is needed: Set the origin of rectangular coordinates system X, Y, Z at the point source, as in Fig. 4, and the three component accelerations recorded at a site are represented as $(a_\gamma, a_\theta, a_\Psi)$, in a spherical coordinates. Their relations with the rectangular one (a_x, a_y, a_z) can be interpreted with orthogonal transformation as follows:

$$\begin{bmatrix} a_\gamma(t) \\ a_\theta(t) \\ a_\Psi(t) \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\Psi & \sin\theta\sin\Psi & \cos\theta \\ \cos\theta\cos\Psi & \cos\theta\sin\Psi & -\sin\theta \\ -\sin\Psi & \cos\Psi & \theta \end{bmatrix} \begin{bmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{bmatrix} \tag{17}$$

Set the seismic record at (a_{SN}, a_{EW}, a_{UD}) the included angle between X positive direction and straight north as γ. Then from the definition of in-plane and anti-plane motions we can get:

$$\begin{bmatrix} a_\gamma(t) \\ a_\theta(t) \\ a_\Psi(t) \end{bmatrix} = \begin{bmatrix} \sin(\gamma-\Psi) & \cos(\gamma-\Psi) & \theta \\ \theta & \theta & -1 \\ \cos(\gamma-\Psi) & \sin(\gamma-\Psi) & \theta \end{bmatrix} \begin{bmatrix} a_{EW}(t) \\ a_{NS}(t) \\ a_{UD}(t) \end{bmatrix} \tag{18}$$

Let $\beta_t = \gamma - \Psi$. Obviously β_t is the angle between the direction of the connected line from the epicenter to the station and the straight north. It can be determined by the epicenter coordinate and the station coordinate. Altering both sides of Eq. (19) by

Fourier transform, we have:

$$\begin{aligned} A_1(\omega) &= \sin\beta_t \cdot A_{EW}(\omega) + \cos\beta_t \cdot A_{NS}(\omega) \\ A_2(\omega) &= -A_{UD}(\omega) \\ A_3(\omega) &= -\cos\beta_t \cdot A_{EW}(\omega) + \sin\beta_t \cdot A_{NS}(\omega) \end{aligned} \quad (19)$$

If the direction of strong seismic record is obtained in the way other than the location of east-west and south-north, it can also be processed in a similar way.

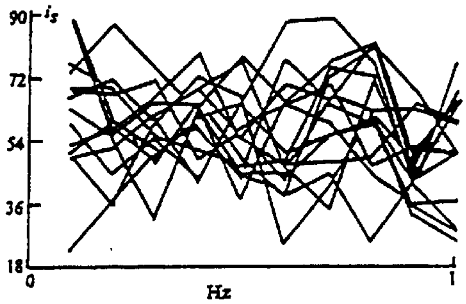

Fig. 5 Variation curve of S-wave incident angle with frequency

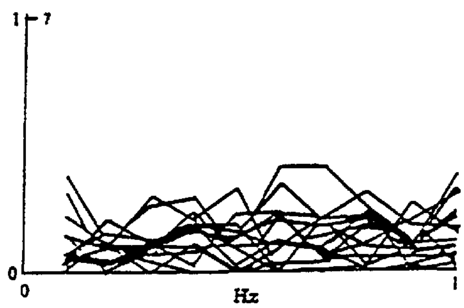

Fig. 6 Variation curve of parameter γ with frequency

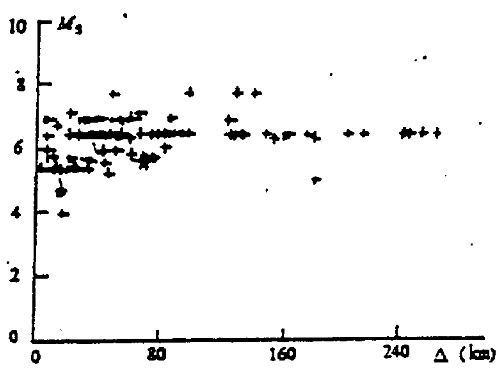

Fig. 7 Relation between magnitude and epicentral distance

4. Statistical Results and Analysis

The estimation of S-wave incident angle is summarized as follows:

1. determine the value β_t and seismic source distance R according to the coordinates of the seismic source and the station.

2. according to Eq. (19) and using the three component records of the strong seismic ground motion, calculate the acceleration spectrum $A_1(\omega)$, $A_2(\omega)$; and filter the records;

3. with $W^0 = \dfrac{U_2^0(\omega)}{U_1^0(\omega)} = \dfrac{A_0 2}{(\omega)} A_1^0(\omega)$ and $W^0 = |W^0| e^{i\theta^0}$, calculate $W^0(\omega)$ and $\theta^0(\omega)$ respectively.

4. inverse S-wave incident angle $i_s(f)$ and $\gamma(f)$ by the method discussed in Section 2.

Figures 5 and 6 demonstrate the S-wave incident angle i_s and variation curve of γ with frequency obtained by the above procedure from 15 seismic records. This paper collected 214 three-component seismic records in western America from 1933 to 1980, with amplitudes 4.0 to 7.7 and epicentral distances 3.5 km to 245 km (Fig. 7). In the frequency range of 0.1-1.0 Hz and with the least squares methods, the statistical equation of the variation of S-wave incident angle with frequency and epicentral distance can be obtained as:

$$\begin{aligned} i_s(\Delta, f) = 25.301 - 41.612 \log_{10} f - 4.791[\log_{10} f]^2 \\ +0.039\Delta + \sigma_1(f) \end{aligned} \quad (20)$$

in which the mean value of random variable $\sigma_1(f)$ is 0, and standard deviation is 14.670. It follows closely the normal distribution, with the unites of i_s, f, Δ and σ as degree, Hz, km and degree respectively. Similarly, the statistical result of $\gamma(\Delta, f)$ is as below:

$$\begin{aligned} \gamma(\Delta, f) = 0.1296 + 0.2504 f + 0.2305 \log_{10} f \\ -0.052[\log_{10} f]^2 + 0.00823\Delta + \sigma_2(f) \end{aligned} \quad (21)$$

where, the mean value of random variable $\sigma_2(f)$ is 0 and the standard deviation is 0.009. It follows closely again the normal distribution, with unites of f, and Δ as above.

As a test of the method proposed in this paper, we have compared the $e^0(f)$ and $\theta^0(f)$ curves of all the true earthquakes with their theoretical $e(f)$ and $\theta(f)$ curves, and found that in the range of 0.1-1.0 Hz frequency band, the calculated $e(f)$ and $\theta(f)$ curves match well the actual $e(f)$ and $\theta(f)$ curves. This indicates that in 0.1-1.0 Hz frequency

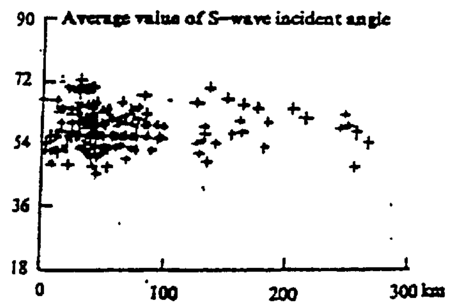

Fig. 8 The curve of the average value of S-wave incident angle with epicentral distance

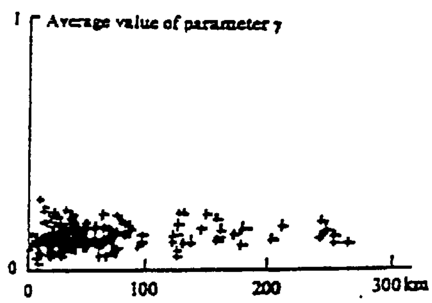

Fig. 9 The curve of the average value of parameter γ with epicentral distance

ranges, the model in this paper can fully interpret the amplitude value spectrum of the station, $W^0(\omega) = |W^0(\omega)|e^{10^0(\omega)}$. It can be seen in the statistical results that epicentral distance has little to do with S-wave incident angle i_s and parameter γ. In order to test this funding, we average S-wave incident angle $i_s(\Delta, f)$ and $\gamma(\Delta, f)$ of each seismic record with frequency within the frequency range of 0.1-1.0 Hz, and obtain $\bar{i}_s(\Delta)$ and $\bar{\gamma}(\Delta)$. The processed result of 214 seismic records is shown in Figs. 8 and 9. In Figs. 8 and 9, it can be seen that the effect of epicentral distance is not obvious on S-wave incident i_s and γ. The calculation shows that in Fig. 8, the average value of S-wave incident angle is 56.78 degrees, and the standard deviation is 6.77°. In Fig. 9, the average value of parameter γ is 0.125, and the standard deviation is 4.27×10^{-2}. As an alternative test of the model in this paper, with the earthquake source theory, it can be calculated that the average value of the directivity radiation factor Ω_p / Ω_s of S-wave and P-wave is 2/3. Let $\beta / \alpha = 1/\sqrt{3}$, and from $\gamma = (\beta / \alpha)^3 \cdot \Omega_p / \Omega_s$, the average value of γ can be obtained as 0.128. This is quite consistent with the average value of 0.125 calculated from the strong seismic records found in this paper.

The research of paper is only fundamental. The problem of the determination of S-wave incident angle in higher frequency range in strong seismic records still awaits further research.

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要旨

均質な弾性半空間媒質内で点震源を仮定しさらに実体波のみを考慮することにより,強震動記録からS波入射角を評価する理論式を求めた。アメリカ西部で観測された214個の強震動記録を用いて,S波入射角,震央距離及び周波数間の統計的関係を求め,S波の平均入射角は震央距離に関係なく約57°となる結果を得た。

キーワード：入射角, S波, 地動, 震央距離, 周波数