

Tsunami Ray-Tracing Utilizing the Bending Method

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Wave ray tracing is a commonly used tool for researches in seismology which provides important information for wave propagation, ray path, and arrival time prediction. In terms of tsunami research, ray tracing is a useful method to assess the tsunami travel path and arrival time and gives us essential information for hazard prevention and mitigation.

There two kinds of ray tracing methods: the shooting method, i.e., the initial value ray tracing, and the bending method, i.e., the boundary value ray tracing. The shooting method gives the direction and the source position to trace the ray path from the source to a specific direction which is numerically stable and fast. However, for problems involve two-point ray tracing, the bending method provides an efficient approach.

Bending Method

The bending method provides an approach for problems involve two-point ray tracing. Numerical method for two-point ray tracing in a heterogeneous isotropic medium has been proposed in 1970s (Pereyra et al., 1980). Pereyra et al. (1980) proposed a formulation of the bending method in Cartesian coordinates, and Koketsu (1991) extended it to spherical coordinates.

Um & Thurber (1987) proposed a pseudo-bending method which provides a fast alternative for the time-consuming bending method (Pereyra et al., 1980). The pseudo-bending method was also extended to spherical coordinates by Koketsu and Sekine (1998) who demonstrates the computational accuracy and efficiency of the pseudo-bending method by

comparing with analytical solutions.

Ray Tracing on Tsunami Researches

Tsunami ray tracing has been discussed since 1980s. Woods and Okal (1987) applied the ray tracing to tsunami propagation in an ocean to reconstruct the tsunami wavefield from the 1960 Chile earthquake. Satake (1988) applied the ray tracing method to tsunami waves to examine bathymetric effects for tsunamis across Pacific Ocean. He demonstrated the focusing and defocusing in some locations and showed the significance for some sources to some coasts. Sansanbata et al. (2018) applied the ray tracing method to dispersive tsunamis and demonstrated that the ray path of dispersive tsunami wave is frequency-dependent.

Although the ray tracing method is widely used in tsunami researches. There are all initial value problems, i.e., they traced the ray paths from the source in a certain area. Here we proposed a newly two-point ray tracing method utilizing the shallow water equation model. Our method provides a fast and stable approach for two-point tsunami ray tracing without heavy computation.

New Tsunami Ray Tracing Method

Assume Assume there is a ray path of minimum travel time from the source (A) to the station (B), \overline{AB} . Then, according to Fermat's principle, this is the least travel time τ_{AB} from A to B. The reverse path from B to A is equal to that from A to B, i.e., $\overline{AB} = \overline{BA}$, and $\tau_{AB} = \tau_{BA} = T$.

The procedure used to find the ray path involves

three steps. In the first step, we calculated the least travel times t_A and t_B from A and B, respectively, to the entire computation domain (Figures a and b). Here, t_A and t_B were calculated using Tsunami Travel Time (TTT, Geoware), which calculates the travel time of the tsunami wave front for all accessible points on the map. The travel time from A to B is given by $t_A(B) = \tau_{AB} = T$, and, similarly, $t_B(A) = T$. Note that $t_A(A) = t_B(B) = 0$. Hence, for $t_A + t_B$ at point A, $t_A(A) + t_B(A) = T$, and and for point B, $t_B(B) + t_A(B) = T$. Therefore, the minimum value of $t_A + t_B$, $\min(t_A + t_B)$, is equal to T , such that $\min(t_A + t_B) = T$ is satisfied at the points along the actual ray path. In the second step, we calculated $t_A + t_B$ for the entire domain, as shown in Figure 3c. In the final step, we traced the path from B to A toward the direction about normal to the contour of t_A along the points with minimum values (white line in Figure c) and obtained the actual path between A and B.

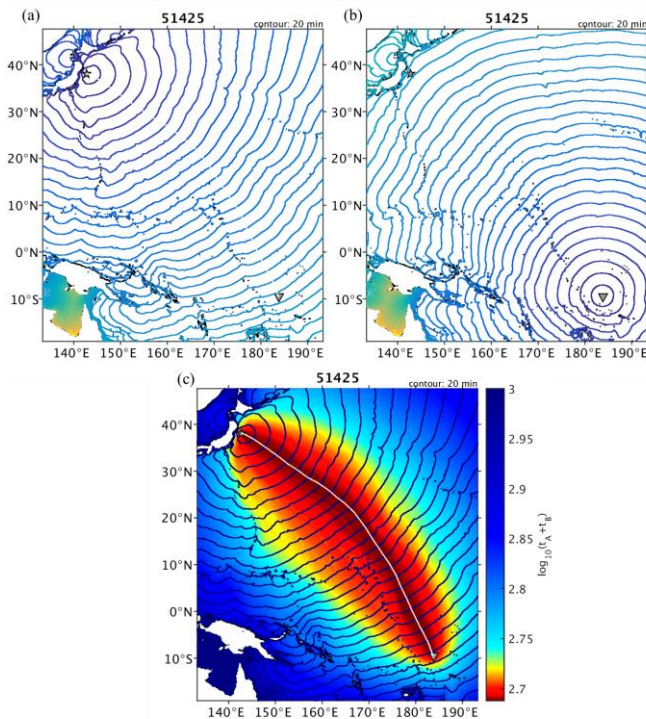


Figure 3. (a) Least travel time from A to the entire domain. (b) Least travel time from B to the entire domain. (c) Map of $\log_{10}(t_A + t_B)$.

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