

A Novel Method for Analysis of Stresses on Hillslopes Considering Climate Change Effect

○Ying-Hsin WU, Eiichi NAKAKITA

Introduction

In recent decades, people around the world have suffered many natural disasters caused by extreme climate events. Under global warming tendency, it is unavoidable to face more severely extreme climate and more corresponding natural hazards. On hillslopes extreme rainfall can trigger massive landslides, e.g. in Nara during Typhoon Talas in 2011. To analyze future hillslope safety under climate change is undoubtedly important and necessary. In this study, we would like to propose a methodology to evaluate hillslope safety considering climate change effects.

Hillslope stability depends on stress equilibrium of soil resistance against driving forces, e.g. groundwater motion and external loading on slope surface. Climate change may bring extreme rainfalls as well as bioenvironmental changes of vegetation on slopes. In other words, extreme rainfall can alter slope saturation and give abundant groundwater flow. Also, vegetation evolution may change surface loading on slopes as well as soil strength due to the effect of root-reinforcement (Sidle, 1992). Therefore, our method shall consider external loading and vegetation effects on hillslope stability.

Methodology

Our methodology comprises four parts: a) soil thickness estimation, b) estimation of vegetation surcharge and root reinforcement, c) stress field analysis, and d) soil failure analysis. Each component is briefly introduced in the following.

Soil thickness estimation

On hillslopes soil depth is usually finite. As exact soil thickness is usually unavailable without field investigation, we use a geomorphological relation of local slope angle (Catani et al., 2010) for approximate thickness estimation.

Vegetation effects on surcharge and soil strength

Being popular for carbon cycle analysis, above ground biomass (AGB) and below ground biomass (BGB) are widely used for estimation of the vegetation weight based on tree allometry (Chave et al., 2005). With AGB from allometric equations the vegetation loading W can be estimated by

$$W = \text{AGB} \times A \quad (1)$$

where A is the area. For root-reinforcement (Wu, 2013), the total cohesion of soil bulk is expressed as

$$c = c_s + c_r \quad (2)$$

where c_s is the original soil cohesion [Pa], and c_r is the root-reinforced one. In most cases, c_r has an exponentially decaying profile in the perpendicular direction of the soil layer (Dupuy et al. 2010).

Evaluation of stress field in the soil layer

A hillslope can be modelled as a poroelastic medium (Biot 1941; Iverson and Reid, 1992). As to consider groundwater effect, we adopt the common effective stress σ'_{ij} as

$$\sigma'_{ij} = \sigma_{ij} + p\delta_{ij}, \quad (3)$$

where p is the pore water pressure [Pa], σ_{ij} is the stress tensor and δ_{ij} is the Kronecker delta function, where indices $(i, j) = \{x, z\}$ as our problem is two-dimensional one in the Cartesian coordinates. With (3) the stress equilibrium of a finite-depth soil layer reads

$$\frac{\partial \sigma'_{xx}}{\partial x} + \frac{\partial \sigma'_{xz}}{\partial z} = \frac{\partial p}{\partial x}, \quad (4)$$

$$\frac{\partial \sigma'_{xz}}{\partial x} + \frac{\partial \sigma'_{zz}}{\partial z} = \rho g + \frac{\partial p}{\partial z}, \quad (5)$$

where σ'_{xx} , σ'_{zz} and σ'_{xz} are effective stresses [Pa], and ρ is the constant bulk density [kg/m³] and g is the gravitational acceleration [m/s²]. Then, for elastic soil, the effective stress compatibility gives

$$(1 - \nu) \nabla^2 (\sigma'_{xx} + \sigma'_{zz}) = \nabla^2 p, \quad (6)$$

where ν is the Poisson ratio of soil bulk, and ∇^2 is the Laplace operator. Finally, it requires an equation for groundwater flow in the soil layer as

$$\nabla^2 (p / \rho_w g + z) = 0, \quad (7)$$

where ρ_w is the water density.

Boundary conditions are no deformation at lateral and bottom boundaries and tractions at soil surface as

$$\left. \begin{aligned} \sigma'_{xx} n_x + \sigma'_{xz} n_z &= 0 \\ \sigma'_{zz} n_z + \sigma'_{xz} n_x &= W \end{aligned} \right\} \text{ at } z = h(x). \quad (8)$$

For saturated condition $p(z=h)=0$ is imposed on soil surface with fluxes at lateral interfaces and no bottom flux. Particularly, infiltration motion caused by extreme rainfall can also be assumed as boundary conditions for solving (7) separately.

As geometry of the problem domain may not be simple, the governing equations, from (3) to (8), are numerically solved by using the finite element method. Then, the method of stress analysis is verified by analytical solutions of problems in a simple geometry.

Soil failure analysis

The Mohr-Coulomb theory (Terzaghi, 1948) is used for slope failure analysis. Using root-reinforced cohesion (2) and solved stresses to find possible failure surface, the yield function F reads

$$F = (\sigma_{xx} - \sigma_{zz})^2 + 4\sigma_{xz}^2 - \sin^2 \phi (\sigma_{xx} + \sigma_{zz} + 2c/\tan \phi)^2, \quad (9)$$

where ϕ is the friction angle [°] of soil bulk, $F < 0$ denotes no failures, and $F = 0$ can provide the

location of failure surface.

Expected results

We shall perform case studies to examine the climate change effects on the stability of hillslopes of different long profiles and vegetation compositions. This method is expected to provide comprehensive analysis of stresses in hillslope, and to assess hillslope stability more accurately. Particularly, this method can also act as a proxy to connect the analysis of hillslope stability with the carbon cycle of forest ecosystem under long-term impact of climate change.

References

- Biot, M.A., General theory of three-dimensional consolidation, *J. Appl. Phys.* 12:155-164, 1941.
- Catani F., Segoni, S., and Falorni, G., An empirical geomorphology-based approach to the spatial prediction of soil thickness at catchment scale, *Water Resour. Res.* 46, W05508, 2010.
- Chave, J., *et al*, Tree allometry and improved estimation of carbon stocks and balance in tropical forests, *Oecologia* 145:87-99, 2005.
- Dupuy L., Gregory P.J., and Bengough G., Root growth models: towards a new generation of continuous approaches, *J. Exp. Bot.* 61(8):2131-2143, 2010.
- Iverson, R.M., and Reid, M.E., Gravity-Driven Groundwater Flow and Slope Failure Potential 1. Elastic Effective-Stress Model, *Water Resour. Res.* 28(3):925-938, 1992.
- Sidle, R.C., A Theoretical Model of the Effects of Timber Harvesting on Slope Stability, *Water Resour. Res.* 28(7):1897-1910, 1992.
- Terzaghi, K., *Theoretical Soil Mechanics*, John Wiley and Son, 1948.
- Wu, T.H., Root reinforcement of soil: review of analytical models, test results, and application of design, *Can. Geotech. J.* 50:259-274, 2013.