

## Derivation of Damage Curves for Quantification of Steel Beam Fractures Induced by Earthquake Loading

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### 1. Introduction

Damage curve is a relationship between strain-based damage index and reduction in beam bending stiffness induced by fracture, from which one can estimate the degree of beam fractures in a steel moment-resisting frame (Kurata et al., 2013; Li et al., 2014). However, the construction of the damage curve required a laborious parametric study simulating various fracture damage in a numerical frame model. This paper presents the derivation of a closed-form expression for damage curves generally applicable for common multi-story multi-bay steel moment-resisting frames.

### 2. Closed-form expression

A damage curve is firstly formulated from a two-story two-bay frame model as a function of various structural parameters. A parametric analysis on the damage curve revealed that the damage curve is primarily dominated by the span-depth ratio of fractured beam. Accordingly, the strain-based damage index ( $DI$ ) is expressed as a function of the reduction of bending stiffness at fractured section in the following simplified formula:

$$DI = \frac{A_1\rho^2 + A_2\rho + A_3}{B_1\rho^2 + B_2\rho + B_3} \times 100\%, \quad (1)$$

*for damage around interior connection*

$$A_1 = -1.803\delta^4 + 5.657\delta^3 - 5.989\delta^2 + 2.671\delta - 0.387,$$

$$A_2 = 1.803\delta^4 - 5.657\delta^3 + 5.989\delta^2 - 2.552\delta,$$

$$A_3 = 0,$$

$$B_1 = \delta^5 - 5.074\delta^4 + 7.57\delta^3 - 6.253\delta^2 + 2.671\delta - 0.387,$$

$$B_2 = -2\delta^5 + 8.345\delta^4 - 7.57\delta^3 + 6.253\delta^2 - 2.552\delta,$$

$$B_3 = \delta^5 - 3.271\delta^4,$$

*for damage around exterior connection*

$$A_1 = -1.502\delta^4 + 4.659\delta^3 - 5.014\delta^2 + 2.28\delta - 0.3278,$$

$$A_2 = 1.502\delta^4 - 4.659\delta^3 + 5.014\delta^2 - 2.161\delta,$$

$$A_3 = 0,$$

$$B_1 = \delta^5 - 4.272\delta^4 + 5.608\delta^3 - 4.791\delta^2 + 2.28\delta - 0.3278,$$

$$B_2 = -2\delta^5 + 7.042\delta^4 - 5.608\delta^3 + 4.791\delta^2 - 2.161\delta,$$

$$B_3 = \delta^5 - 2.77\delta^4,$$

where  $\rho$  is reduction in bending stiffness at fractured section;  $\delta$  is the span-depth ratio of the fractured beam.

Given the damage index  $DI$ , the reduction of bending stiffness at fractured section is expressed as:

$$\rho = \frac{-(B_2(DI) - A_2) - \sqrt{(B_2(DI) - A_2)^2 - 4(B_1(DI) - A_1)(B_3(DI) - A_3)}}{2(B_1(DI) - A_1)}, \quad (2)$$

### 3. Verification

A series of ambient vibration tests on a steel frame test bed (Figure 1) were conducted to experimentally verify the closed-form expression. Fracture damage with four levels of damage extent was simulated (Table 1). Four damage cases, i.e. Case (a) to Case (d), with different fracture position were considered. In Case (a) and Case (b), fractures were at the removable connections B1 and B5 near the exterior beam-column connections, while fractures were at the removable connections B2 and B6 near the inner beam-column connections in Case (c) and Case (d) (see Figure 1(b)). The fractured beam in four cases had the same span-depth ratio.

Figure 2 shows the damage indices obtained from the closed-form expression and experimental investigations. Small discrepancy between two curves illustrated the effectiveness of the expression in constructing damage curves for beam fractures for any multi-story multi-bay steel moment-resisting frames. Figure 3 shows reduced bending stiffness evaluated from Equation (2) using experimental damage index. When the evaluated values of reduced bending stiffness was compared with the exact values (i.e., computed from the measured sectional properties), the

absolute difference was about 9% for damage L1 and L2, and 3% for damage L3 and L4.

The presented analytical expression facilitates the use of the strain based damage quantification of steel beam fractures in structural health monitoring of steel buildings as it dramatically reduces efforts in constructing numerical models for deriving damage curves.

#### 4. References

Kurata M., Li X., Fujita K., and Yamaguchi M. (2013): Piezoelectric dynamic strain monitoring for detecting local seismic damage in steel buildings. *Smart Mater. Struct.*, 22, 115002. DOI: 10.1088/0964-1726/22/11/115002.

Li X., Kurata M., and Nakashima M. (2014): Evaluating damage extent of fractured beams in steel moment-resisting frames using dynamic strain responses. *Earthquake Engng Struct. Dyn.*. DOI: 10.1002/eqe.2536.

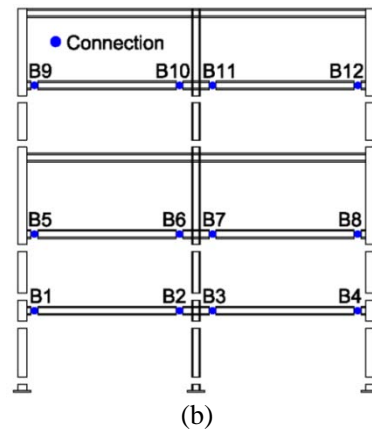
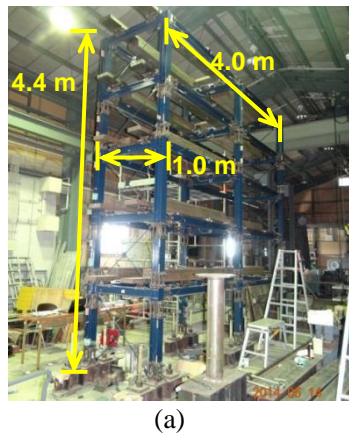


Fig. 1. Steel frame test bed: (a) overview; (b) beam connections.

Table 1. Damage patterns.

Damage pattern	Target of simulation	Reduction of $EI_x$ (%)
L1	Fracture of half bottom flange	21.9
L2	Fracture of whole bottom flange	53.4
L3	Fracture of bottom flange and one-quarter web	79.4
L4	Fracture of bottom flange and half web	93.6

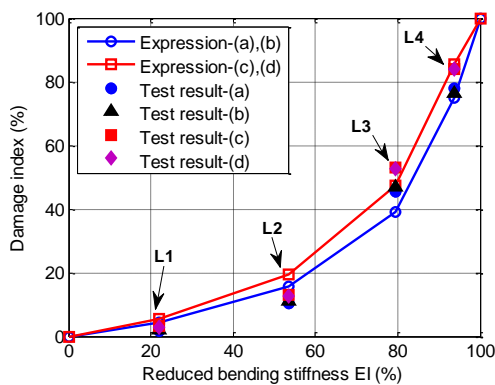


Fig. 2. Comparison of damage index obtained from closed-form expression and experimental investigation.

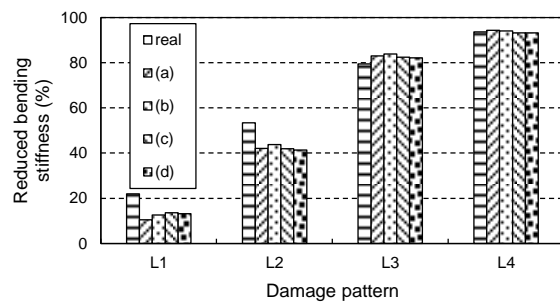


Fig. 3. Reduced bending stiffness evaluated from closed-form expression using experimental damage index.